Chapter - Probability

Topic-1: Multiplication Theorem on Probability, Independent Events, Conditional Probability, Baye's Theorem

1 MCQs with One Correct Answer

1. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is [Adv. 2024]

   (a) $\frac{1}{12}$ (b) $\frac{1}{7}$

   (c) $\frac{5}{7}$ (d) $\frac{5}{12}$

2. Let $X = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is [Adv. 2023]

   (a) $\frac{71}{220}$ (b) $\frac{73}{220}$

   (c) $\frac{79}{220}$ (d) $\frac{83}{220}$

3. Consider 4 boxes, where each box contains 3 red balls and 2 blue balls. Assume that all 20 balls are distinct. In how many different ways can 10 balls be chosen from these 4 boxes so that from each box at least one red ball and one blue ball are chosen ? [Adv. 2022]

   (a) 21816 (b) 85536 (c) 12096 (d) 156816

4. Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements.

   Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

   Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements.

   Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is [Adv. 2021]

   (a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{5}$

5. Let C_1 and C_2 be two biased coins such that the probabilities of getting head in a single toss are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Suppose α is the number of heads that appear when C_1 is tossed twice, independently, and suppose β is the number of heads that appear when C_2 is tossed twice, independently. Then the probability that the roots of the quadratic polynomial $x^2 - \alpha x + \beta$ are real and equal, is [Adv. 2020]

   (a) $\frac{40}{81}$ (b) $\frac{20}{81}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

6. Three randomly chosen non–negative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is [Adv. 2017]

(a) $\frac{36}{55}$ (b) $\frac{6}{11}$ (c) $\frac{1}{2}$ (d) $\frac{5}{11}$

7. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective given that it is produced in plant T_1) = 10P (computer turns out to be defective given that it is produced in plant T_2), where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is [Adv. 2016]

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$ (c) $\frac{78}{93}$ (d) $\frac{75}{83}$

8. Four fair dice D_1, D_2, D_3 and D_4; each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is [2012]

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$ (c) $\frac{125}{216}$ (d) $\frac{127}{216}$

9. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is [2010]

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{20}{23}$ (d) $\frac{9}{20}$

10. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is [2010]

(a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9}$ (d) $\frac{1}{36}$

11. An experiment has 10 equally likely outcomes. Let A and B be non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is [2008]

(a) 2, 4 or 8 (b) 3, 6 or 9 (c) 4 or 8 (d) 5 or 10

12. Let E^c denote the complement of an event E. Let E, F, G be pairwise independent events with $P(G) > 0$ and $P(E \cap F \cap G) = 0$. Then $P(E^c \cap F^c | G)$ equals [2007 -3 marks]

(a) $P(E^c) + P(F^c)$ (b) $P(E^c) - P(F^c)$

(c) $P(E^c) - P(F)$ (d) $P(E) - P(F^c)$

13. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife given that each American man is seated adjacent to his wife is [2007 -3 marks]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

14. A six faced fair dice is thrown until 1 comes, then the probability that 1 comes in even no. of trials is [2005S]

(a) 5/11 (b) 5/6 (c) 6/11 (d) 1/6

15. Two numbers are selected randomly from the set $S = \{1, 2, 3, 4, 5, 6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4 is [2003S]

(a) 1/15 (b) 14/15 (c) 1/5 (d) 4/5

16. If the integers m and n are chosen at random from 1 to 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals [1999 - 2 Marks]

(a) 1/4 (b) 1/7 (c) 1/8 (d) 1/49

17. Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral, equals [1995S]

(a) 1/2 (b) 1/5 (c) 1/10 (d) 1/20

18. Let A, B, C be three mutually independent events. Consider the two statements S_1 and S_2

S_1 : A and $B \cup C$ are independent

S_2 : A and $B \cap C$ are independent

Then, [1994]

(a) Both S_1 and S_2 are true

(b) Only S_1 is true

(c) Only S_2 is true

(d) Neither S_1 nor S_2 is true

19. An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained, the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is then: [1993 - 1 Mark]

(a) 16/81 (b) 1/81 (c) 80/81 (d) 65/81

20. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting, points 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is [1992 - 2 Marks]

(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

21. Fifteen coupons are numbered 1, 2 ..... 15, respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9, is [1983 - 1 Mark]

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) none of these

22. If A and B are two events such that $P(A) > 0$, and $P(B) \neq 1$, then $P\left(\frac{\overline{A}}{\overline{B}}\right)$ is equal to [1982 - 2 Marks]

(a) $1 - P\left(\frac{A}{B}\right)$ (b) $1 - P\left(\frac{\overline{A}}{B}\right)$

(c) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (d) $\frac{P(\overline{A})}{P(\overline{B})}$

(Here $\overline{A}$ and $\overline{B}$ are complements of A and B respectively).

23. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiment are performed. The probability that the event A happens at least once is [1980]

(a) 0.936 (b) 0.784
(c) 0.904 (d) none of these

24. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is [1980]
(a) 0.39 (b) 0.25
(c) 0.11 (d) none of these

25. Two fair dice are tossed. Let x be the event that the first die shows an even number and y be the event that the second die shows an odd number. The two events x and y are: [1979]
(a) Mutually exclusive
(b) Independent and mutually exclusive
(c) Dependent
(d) None of these.

2 Integer Value Answer/ Non-Negative Integer

26. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let W_i, G_i and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 | W_1 \cap G_2) = \frac{2}{9}$, then N equals ______. [Adv. 2024]

27. Let X be the set of all five digit numbers formed using 1,2,2,2,4,4,0. For example, 22240 is in X while 02244 and 44422 are not in X. Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to [Adv. 2023]

28. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(\alpha\ 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1). [Adv. 2013]

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$

3 Numeric/New Stem Based Questions

Question Stem for Question Nos. 29 and 30

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, ..., 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

29. The value of $\frac{625}{4} p_1$ is ______. [Adv. 2021]

30. The value of $\frac{125}{4} p_2$ is ______. [Adv. 2021]

31. Let S be the sample space of all 3×3 matrices with entries from the set $\{0,1\}$. Let the events E_1 and E_2 be given by $E_1 = \{A \in S : det\ A = 0\}$ and $E_2 = \{A \in S$: sum of entries of A is 7$\}$.
If a matrix is chosen at random from S, then the conditional probability $P(E_1/E_2)$ equals ______ [Adv. 2019]

32. Let $|X|$ denote the number of elements in a set X. Let $S = \{1, 2, 3, 4, 5, 6\}$ be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \le |B| < |A|$, equals ______. [Adv. 2019]

4 Fill in the Blanks

33. If two events A and B are such that $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$, then $P(B/(A \cup B^c)) =$ .... [1994 - 2 Marks]

34. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. If A and B are independent events then $P(B) =$ .... [1990 - 2 Marks]

35. Urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. If one ball is now drawn at random from urn A, the probability that it is found to be red is................ [1988 - 2 Marks]

36. If $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is .... [1986 - 2 Marks]

37. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability .... [1985 - 2 Marks]

5 True / False

38. If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is 0.5. [1989 - 1 Mark]

39. If the letters of the word "Assassin" are written down at random in a row, the probability that no two S's occur together is 1/35 [1983 - 1 Mark]

6 MCQs with One or More than One Correct Answer

40. There are three bags B_1, B_2, and B_3. The bag B_1 contains 5 red and 5 green balls, B_2 contains 3 red and 5 green balls, and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10}, \frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct? [Adv. 2019]
(a) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$
(b) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(c) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

(d) Probability that the chosen ball is green equals $\frac{39}{80}$

41. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{2}{5}$. Then [Adv. 2017]

(a) $P(Y) = \frac{4}{15}$ (b) $P(X'|Y) = \frac{1}{2}$

(c) $P(X \cap Y) = \frac{1}{5}$ (d) $P(X \cup Y) = \frac{2}{5}$

42. Let X and Y be two events such that $P(X|Y) = \frac{1}{2}$, $P(Y/X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is (are) correct? [2012]

(a) $P(X \cup Y) = \frac{2}{3}$

(b) X and Y are independent

(c) X and Y are not independent

(d) $P(X^c \cap Y) = \frac{1}{3}$

43. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is(are) true? [2012]

(a) $P\left[X_1^c | X\right] = \frac{3}{16}$

(b) P [Exactly two engines of the ship are functioning $|X] = \frac{7}{8}$

(c) $P[X|X_2] = \frac{5}{16}$

(d) $P[X|X_1] = \frac{7}{16}$

44. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then [2011]

(a) $P(E) = \frac{4}{5}, P(F) = \frac{3}{5}$ (b) $P(E) = \frac{1}{5}, P(F) = \frac{2}{5}$

(c) $P(E) = \frac{2}{5}, P(F) = \frac{1}{5}$ (d) $P(E) = \frac{3}{5}, P(F) = \frac{4}{5}$

45. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two. Which of the following relations are true? [1999 - 3 Marks]

(a) $p+m+c=19/20$ (b) $p+m+c=27/20$

(c) $pmc=1/10$ (d) $pmc=1/4$

46. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals [1998 - 2 Marks]

(a) 1/2 (b) 7/15

(c) 2/15 (d) 1/3

47. If E and F are events with $P(E) \leq P(F)$ and $P(E \cap F) > 0$, then [1998 - 2 Marks]

(a) occurrence of $E \Rightarrow$ occurrence of F

(b) occurrence of $F \Rightarrow$ occurrence of E

(c) non-occurrence of $E \Rightarrow$ non-occurrence of F

(d) none of the above implications holds

48. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is [1998 - 2 Marks]

(a) 1/3 (b) 1/6 (c) 1/2 (d) 1/4

49. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then [1998 - 2 Marks]

(a) $P(E/F) + P(\overline{E}/F) = 1$

(b) $P(E/F) + P(E/\overline{F}) = 1$

(c) $P(\overline{E}/F) + P(E/\overline{F}) = 1$

(d) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

50. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is [1998 - 2 Marks]

(a) 13/32 (b) 1/4

(c) 1/32 (d) 3/16

51. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ then [1995S]

(a) $P(B/A) = P(B) - P(A)$

(b) $P(A' - B') = P(A') - P(B')$

(c) $P(A \cup B)' = P(A')\, P(B')$

(d) $P(A/B) = P(A)$

52. E and F are two independent events. The probability that both E and F happen is 1/12 and the probability that neither E nor F happens is 1/2. Then, [1993 - 2 Marks]

(a) $P(E) = 1/3, P(F) = 1/4$

(b) $P(E) = 1/2, P(F) = 1/6$

(c) $P(E) = 1/6, P(F) = 1/2$

(d) $P(E) = 1/4, P(F) = 1/3$

53. For any two events A and B in a sample space [1991 - 2 Marks]

(a) $P(A/B) \geq \frac{P(A)+P(B)-1}{P(B)}, P(B) \neq 0$ is always true

(b) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$ does not hold

(c) $P(A \cup B) = 1 - P(\overline{A})\,P(\overline{B})$, if A and B are independent

(d) $P(A \cup B) = 1 - P(\overline{A})\,P(\overline{B})$, if A and B are disjoint

54. If E and F are independent events such that $0 < P(E) < 1$ and $0 < P(F) < 1$, then [1989 - 2 Marks]
(a) E and F are mutually exclusive
(b) E and F^c (the complement of the event F) are independent
(c) E^c and F^c are independent
(d) $P(E|F) + P(E^c|F) = 1$.

55. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\overline{A}) + P(\overline{B})$ is [1987 - 2 Marks]
(a) 0.4 (b) 0.8 (c) 1.2 (d) 1.4
(e) none
(Here $\overline{A}$ and $\overline{B}$ are complements of A and B, respectively).

56. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II and III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then [1986 - 2 Marks]
(a) $p = q = 1$ (b) $p = q = \frac{1}{2}$
(c) $p = 1, q = 0$ (d) $p = 1, q = \frac{1}{2}$
(e) none of these

8 Comprehension/Passage Based Questions

PASSAGE - 1

There are five students S_1, S_2, S_3, S_4 and S_5 in a music class and for them there are five seats R_1, R_2, R_3, R_4 and R_5 arranged in a row, where initially the seat R_i is allotted to the student S_i, $i = 1, 2, 3, 4, 5$. But, on the examination day, the five students are randomly allotted the five seats. [Adv. 2018]

57. The probability that, on examination day, the student S_1 gets the previously allotted seat R_1, and NONE of the remaining students gets the seat previously allotted to him/her is
(a) $\frac{3}{40}$ (b) $\frac{1}{8}$ (c) $\frac{7}{40}$ (d) $\frac{1}{5}$

58. For $i = 1, 2, 3, 4$, let T_i denote the event that the students S_i and S_{i+1} do NOT sit adjacent to each other on the day of the examination. Then, the probability of the event $T_1 \cap T_2 \cap T_3 \cap T_4$ is
(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{7}{60}$ (d) $\frac{1}{5}$

PASSAGE - 2

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II. [Adv. 2015]

59. One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are)
(a) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$
(b) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$
(c) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$
(d) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

60. A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is(are)
(a) $n_1 = 4$ and $n_2 = 6$
(b) $n_1 = 2$ and $n_2 = 3$
(c) $n_1 = 10$ and $n_2 = 20$
(d) $n_1 = 3$ and $n_2 = 6$

PASSAGE - 3

Box 1 contains three cards bearing numbers 1, 2, 3; box 2 contains five cards bearing numbers 1, 2, 3, 4, 5; and box 3 contains seven cards bearing numbers 1, 2, 3, 4, 5, 6, 7. A card is drawn from each of the boxes. Let x_i be number on the card drawn from the i^{th} box, $i = 1, 2, 3$. [Adv. 2014]

61. The probability that $x_1 + x_2 + x_3$ is odd, is
(a) $\frac{29}{105}$ (b) $\frac{53}{105}$ (c) $\frac{57}{105}$ (d) $\frac{1}{2}$

62. The probability that x_1, x_2, x_3 are in an arithmetic progression, is
(a) $\frac{9}{105}$ (b) $\frac{10}{105}$ (c) $\frac{11}{105}$ (d) $\frac{7}{105}$

PASSAGE - 4

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls. [Adv. 2013]

63. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is
(a) $\frac{82}{648}$ (b) $\frac{90}{648}$ (c) $\frac{558}{648}$ (d) $\frac{566}{648}$

64. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is
(a) $\frac{116}{181}$ (b) $\frac{126}{181}$ (c) $\frac{65}{181}$ (d) $\frac{55}{181}$

PASSAGE - 5

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2. [2011]

65. The probability of the drawn ball from U_2 being white is
(a) $\frac{13}{30}$ (b) $\frac{23}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

66. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(a) $\frac{17}{23}$ (b) $\frac{11}{23}$ (c) $\frac{15}{23}$ (d) $\frac{12}{23}$

9 Assertion and Reason/ Statement Type Questions

67. Consider the system of equations $ax + by = 0$; $cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$

STATEMENT - 1 : The probability that the system of equations has a unique solution is $\frac{3}{8}$. and

STATEMENT - 2 : The probability that the system of equations has a solution is 1. [2008]

(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explaination for STATEMENT - 1
(c) STATEMENT - 1 is True, STATEMENT - 2 is False
(d) STATEMENT - 1 is False, STATEMENT - 2 is True

68. Let $H_1, H_2, ..., H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, ..., n$. Let E be any other event with $0 < P(E) < 1$.

STATEMENT-1 :
$P(H_i | E) > P(E | H_i) . P(H_i)$ for $i = 1, 2, ..., n$ because

STATEMENT-2 : $\sum_{i=1}^{n} P(H_i) = 1$. [2007 -3 marks]

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

10 Subjective Problems

69. A person goes to office either by car, scooter, bus or train, the probability of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$ respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}$ and $\frac{1}{9}$ respectively. Given that he reached office in time, then what is the probability that he travelled by a car. [2005 - 2 Marks]

70. A box contains 12 red and 6 white balls. Balls are drawn from the box one at a time without replacement. If in 6 draws there are at least 4 white balls, find the probability that exactly one white is drawn in the next two draws. (binomial coefficients can be left as such) [2004 - 4 Marks]

71. A and B are two independent events. C is event in which exactly one of A or B occurs. Prove that

$P(C) \geq P(A \cup B) P(\bar{A} \cap \bar{B})$ [2004 - 2 Marks]

72. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit then find the probability that B hits the target and C does not. [2003 - 2 Marks]

73. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1[st] exam is p. If he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify. [2003 - 2 Marks]

74. A box contains N coins, m of which are fair and the rest are biased. The probability of getting a head when a fair coin is tossed is 1/2, while it is 2/3 when a biased coin is tossed. A coin is drawn from the box at random and is tossed twice. The first time it shows head and the second time it shows tail. What is the probability that the coin drawn is fair? [2002 - 5 Marks]

75. An unbiased die, with faces numbered 1, 2, 3, 4, 5, 6, is thrown n times and the list of n numbers showing up is noted. What is the probability that, among the numbers 1, 2, 3, 4, 5, 6, only three numbers appear in this list? [2001 - 5 Marks]

76. An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. What is the probability that the ball drawn now is white? [2001 - 5 Marks]

77. A coin has probability p of showing head when tossed. It is tossed n times. Let p_n denote the probability that no two (or more) consecutive heads occur. Prove that $p_1=1$, $p_2 = 1-p^2$ and $p_n=(1-p)$. $p_{n-1} + p(1-p)\, p_{n-2}$ for all $n \geq 3$. [2000 - 5 Marks]

78. Eight players $P_1, P_2, ..., P_8$ play a knock-out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, what is the probability that the player P_4 reaches the final? [1999 - 10 Marks]

79. Three players, A, B and C, toss a coin cyclically in that order (that is $A, B, C, A, B, C, A, B, ...$) till a head shows. Let p be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and C gets the first head. Prove that $\beta = (1 - p)\,\alpha$. Determine α, β and γ (in terms of p). [1998 - 8 Marks]

80. If p and q are chosen randomly from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, with replacement, determine the probability that the roots of the equation $x^2 + px + q = 0$ are real. [1997 - 5 Marks]

81. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats? Now, if all the seating arrangements are equally likely, what is the probability of 3 girls sitting together in a back row on adjacent seats? [1996 - 5 Marks]

82. An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4,.....12 is picked and the number on the card is noted. What is the probability that the noted number is either 7 or 8? [1994 - 5 Marks]

83. A lot contains 50 defective and 50 non defective bulbs. Two bulbs are drawn at random, one at a time, with replacement. The events A, B, C are defined as

A = (the first bulb is defective) [1992 - 6 Marks]

B = (the second bulb is non-defective)

C = (the two bulbs are both defective or both non defective)

Determine whether

(i) A, B, C are pairwise independent

(ii) A, B, C are independent

84. In a test an examine either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he make a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it, is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it. [1991 - 4 Marks]

85. A is a set containing n elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen at random. Find the probability that P and Q have no common elements. [1990 - 5 Marks]

86. A box contains 2 fifty paise coins, 5 twenty five paise coins and a certain fixed number N (≥ 2) of ten and five paise coins. Five coins are taken out of the box at random. Find the probability that the total value of these 5 coins is less than one rupee and fifty paise. [1988 - 3 Marks]

87. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelth testing. [1986 - 5 Marks]

88. In a multiple-choice question there are four alternative answers, of which one or more are correct. A candidate will get marks in the question only if he ticks the correct answers. The candidate decides to tick the answers at random, if he is allowed upto three chances to answer the questions, find the probability that he will get marks in the questions. [1985 - 5 Marks]

89. In a certain city only two newspapers A and B are published, it is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. It is also known that 30% of those who read A but not B look into advertisements and 40% of those who read B but not A look into advertisements while 50% of those who read both A and B look into advertisements. What is the percentage of the population that reads an advertisement? [1984 - 4 Marks]

90. A, B, C are events such that [1983 - 2 Marks]

$P(A) = 0.3, P(B) = 0.4, P(C) = 0.8$

$P(AB) = 0.08, P(AC) = 0.28; \quad P(ABC) = 0.09$

If $P(A \cup B \cup C) \geq 0.75$, then show that $P(BC)$ lies in the interval $0.23 \leq x \leq 0.48$

91. Cards are drawn one by one at random from a well -shuffled full pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then show that

$$P_r\{N = n\} = \frac{(n-1)(52-n)(51-n)}{50 \times 49 \times 17 \times 13} \text{ where } 2 \leq n \leq 50$$

[1983 - 3 Marks]

92. A and B are two candidates seeking admission in IIT. The probability that A is selected is 0.5 and the probability that both A and B are selected is atmost 0.3. Is it possible that the probability of B getting selected is 0.9? [1982 - 2 Marks]

93. An anti -aircraft gun can take a maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane ? [1981 - 2 Marks]

94. Balls are drawn one-by-one without replacement from a box containing 2 black, 4 white and 3 red balls till all the balls are drawn. Find the probability that the balls drawn are in the order 2 black, 4 white and 3 red. [1978]

Topic-2: Random Variables, Probability Distribution, Bernoulli Trails, Binomial Distribution, Poisson Distribution

1 MCQs with One Correct Answer

1. Consider an experiment of tossing a coin repeatedly until the outcomes of two consecutive tosses are same. If the probability of a random toss resulting in head is $\frac{1}{3}$, then the probability that the experiment stops with head is [Adv. 2023]

(a) $\frac{1}{3}$ (b) $\frac{5}{21}$ (c) $\frac{4}{21}$ (d) $\frac{2}{7}$

2. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is [Adv. 2014]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

3. The probability of India winning a test match against west Indies is 1/2. Assuming independence from match to match the probability that in a 5 match series India's second win occurs at third test is [1995S]

(a) 1/8 (b) 1/4 (c) 1/2 (d) 2/3

4. One hundred identical coins, each with probability, p, of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is [1988 - 2 Marks]

(a) 1/2 (b) 49/101
(c) 50/101 (d) 51/101.

5. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is [1984 - 2 Marks]
(a) 5/64 (b) 27/32 (c) 5/32 (d) 1/2

2 Integer Value Answer/ Non-Negative Integer

6. Let X be a random variable, and let $P(X=x)$ denote the probability that X takes the value x. Suppose that the points $(x, P(X=x))$, $x=0,1,2,3,4$ lie on a fixed straight line in the xy-plane, and $P(X=x)=0$ for all $x \in \mathbb{R}-\{0,1,2,3,4\}$.

If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is ______. [Adv. 2024]

7. The probability that a missile hits a target successfully is 0.75. In order to destroy the target completely, at least three successful hits are required. Then the minimum number of missiles that have to be fired so that the probability of completely destroying the target is **NOT** less than 0.95, is _____ [Adv. 2020]

8. The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96, is [Adv. 2015]

3 Numeric/ New Stem Based Questions

9. Let p_i be the probability that a randomly chosen point has i many friends, $i = 0,1,2,3,4$. Let X be a random variable such that for $i = 0,1,2,3,4$, the probability $P(X = i) = p_i$. Then the value of 7E(X) is [Adv. 2023]

10. Two distinct points are chosen randomly out of the points $A_1, A_2, \ldots\ldots A_{49}$. Let p be the probability that they are friends. Then the value of 7 p is [Adv. 2023]

11. Two fair dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled together and the sum of the numbers on the faces is observed. This process is repeated till the sum is either a prime number or a perfect square. Suppose the sum turns out to be a perfect square before it turns out to be a prime number. If p is the probability that this perfect square is an odd number, then the value of $14p$ is ____ [Adv. 2020]

4 Fill in the Blanks

12. $P(A \cup B) = P(A \cap B)$ if and only if the relation between $P(A)$ and $P(B)$ is .... [1985 - 2 Marks]

6 MCQs with One or More than One Correct Answer

13. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is [Adv. 2013]
(a) $\frac{235}{256}$ (b) $\frac{21}{256}$
(c) $\frac{3}{256}$ (d) $\frac{253}{256}$

14. A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals [1998 - 2 Marks]
(a) 1/2 (b) 1/32
(c) 31/32 (d) 1/5

8 Comprehension/Passage Based Questions

PASSAGE - 1

A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. [2009]

15. The probability that $X = 3$ equals
(a) $\frac{25}{216}$ (b) $\frac{25}{36}$
(c) $\frac{5}{36}$ (d) $\frac{125}{216}$

16. The probability that $X \geq 3$ equals
(a) $\frac{125}{216}$ (b) $\frac{25}{36}$
(c) $\frac{5}{36}$ (d) $\frac{25}{216}$

17. The conditional probability that $X \geq 6$ given $X > 3$ equals
(a) $\frac{125}{216}$ (b) $\frac{25}{216}$
(c) $\frac{5}{36}$ (d) $\frac{25}{36}$

PASSAGE - 2

There are n urns, each of these contain $n+1$ balls. The ith urn contains i white balls and $(n+1-i)$ red balls. Let u_i be the event of selecting ith urn, $i = 1, 2, 3 \ldots\ldots\ldots, n$ and w the event of getting a white ball. [2006 - 5M, –2]

18. If $P(u_i) \propto i$, where $i = 1, 2, 3, \ldots\ldots, n$, then $\lim_{n\to\infty} P(w) =$
(a) 1 (b) 2/3
(c) 3/4 (d) 1/4

19. If $P(u_i) = c$, (a constant) then $P(u_n/w) =$
(a) $\frac{2}{n+1}$ (b) $\frac{1}{n+1}$
(c) $\frac{n}{n+1}$ (d) $\frac{1}{2}$

20. Let $P(u_i) = \frac{1}{n}$, if n is even and E denotes the event of choosing even numbered urn, then the value of $P(w/E)$ is

(a) $\dfrac{n+2}{2n+1}$ (b) $\dfrac{n+2}{2(n+1)}$

(c) $\dfrac{n}{n+1}$ (d) $\dfrac{1}{n+1}$

10 Subjective Problems

21. Numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02....., 99 with replacement. An event E occurs if only if the product of the two digits of a selected number is 18. If four numbers are selected, find probability that the event E occurs at least 3 times. **[1993 - 5 Marks]**

22. Suppose the probability for A to win a game against B is 0.4. If A has an option of playing either a "best of 3 games" or a "best of 5 games" match against B, which option should he choose so that the probability of his winning the match is higher? (No game ends in a draw). **[1989 - 5 Marks]**

23. A man takes a step forward with probability 0.4 and backwards with probability 0.6 Find the probability that at the end of eleven steps he is one step away from the starting point. **[1987 - 3 Marks]**

Answer Key

Topic-1 : Multiplication Theorem on Probability, Independent Events, Conditional Probability, Baye's Theorem

1. (c) 2. (b) 3. (a) 4. (a) 5. (b) 6. (b) 7. (c) 8. (a) 9. (c) 10. (c)
11. (d) 12. (c) 13. (c) 14. (a) 15. (d) 16. (a) 17. (c) 18. (a) 19. (a) 20. (b)
21. (d) 22. (c) 23. (b) 24. (a) 25. (d) 26. (11) 27. (31) 28. (6) 29. (76.25) 30. (24.50) 31. (0.50) 32. (422) 33. $\frac{1}{4}$ 34. $\frac{5}{7}$ 35. $\frac{32}{55}$ 36. $\frac{1}{3} \le p \le \frac{1}{2}$ 37. $\frac{1}{9}$ 38. (False) 39. (False) 40. (b, d) 41. (a, b) 42. (a, b) 43. (b, d) 44. (a, d) 45. (b, c) 46. (b) 47. (d) 48. (b) 49. (a, d) 50. (a) 51. (c, d) 52. (a, d) 53. (a, c) 54. (b, c, d) 55. (c) 56. (c) 57. (a) 58. (c) 59. (a, b) 60. (c, d) 61. (b) 62. (c) 63. (a) 64. (d) 65. (b) 66. (d) 67. (b) 68. (d)

Topic-2 : Random Variables, Probability Distribution, Bernoulli Trails, Binomial Distribution, Poisson Distribution

1. (b) 2. (a) 3. (b) 4. (d) 5. (c) 6. (42) 7. (6) 8. (8) 9. (24.00) 10. (0.50)
11. (75) 12. [P(A∩B)] 13. (a) 14. (a) 15. (a) 16. (b) 17. (d) 18. (b) 19. (a)
20. (b)

Hints & Solutions

Topic-1: Multiplication Theorem on Probability, Independent events, Conditional Probability, Baye's Theorem

1. **(c)** Let

$E_1 \rightarrow$ Knows the answer

$E_2 \rightarrow$ Guesses the answer

$F \rightarrow$ Answer will correct

Let $P(E_1) = k$ then $P(E_2) = 1 - k$

Given that $P(F/E_2) = \frac{1}{2}$, $P(E_2/F) = \frac{1}{6}$

and $P(F/E_1) = 1$

$$\therefore P(E_2/F) = \frac{P(F/E_2).P(E_2)}{P(F/E_1).P(E_1) + P(F/E_2).P(E_2)}$$

$$= \frac{(1-k)\left(\frac{1}{2}\right)}{(1-k)\left(\frac{1}{2}\right) + k(1)} = \frac{1}{6}$$

$$\Rightarrow (3-3k) = \frac{1}{2} + \frac{k}{2} \Rightarrow \frac{5}{2} = \frac{7k}{2} \Rightarrow k = \frac{5}{7}$$

2. **(b)** Given that

$$\overline{X} = \left\{(x, y) \in Z \times Z : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$$

Let $\frac{x^2}{8} + \frac{y^2}{20} = 1$ ...(i)

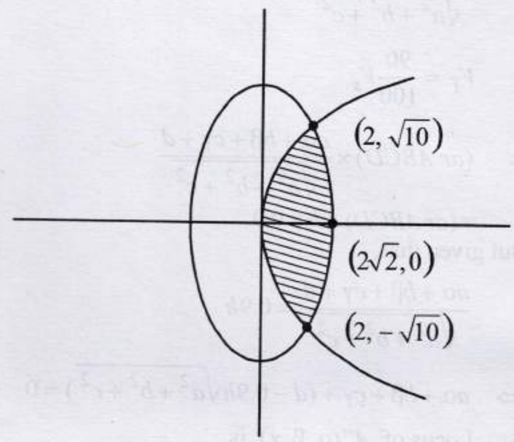
and $y^2 = 5x$ .....(ii)

Solving equation (i) and (ii) we get

$x = 2$ and $y = \pm\sqrt{10}$

The points inside region

x = {(1, 1), (1, 0), (1, –1), (1, 2), (1, –2) (2, 1), (2, 2), (2, 3), (2, 0), (2, –1) (2, –2), (2, –3)}.



Total number of ways to select three points

$S = {}^{12}C_3 = 220$

For favourable events

Selecting 3 points in which 2 points are either x = 1 or x = 2 but distance between then is even.

Case 1: Triangles with base $2 = 3 \times 7 + 5 \times 5 = 46$

Case 2: Triangles with base $4 = 1 \times 7 + 3 \times 5 = 22$

Case 3: Triangles with base $6 = 1 \times 5$

$$\therefore P(E) = \frac{46 + 22 + 5}{220} = \frac{73}{220}$$

3. **(a)**

	Box-1	Box-2	Box-3	Box-4
	3R 2B	3R 2B	3R 2B	3R 2B
Case I	4 Balls	2 Balls	2 Balls	2 Balls
Case II	3 Balls	3 Balls	2 Balls	2 Balls
	2R 1B 2B 1R	2R 1B 2B 1R		

${}^4C_1$[3 Red 1 Blue 2 Red 2 Blue] + case II

$$= {}^4C_1\left[{}^3C_3 . {}^2C_1 + {}^3C_2 . {}^2C_2\right]\left({}^3C_1 {}^2C_1\right)^3 + {}^4C_2\left[{}^3C_2 {}^2C_1 + {}^3C_1 {}^2C_2\right]^2\left[{}^3C_1 {}^2C_1\right]^2$$

$$= 4(5)(6)^3 + 6(3\times 2+3)^2 (6)^2 = 4320 + 17496$$

$= 21816$ (option a)

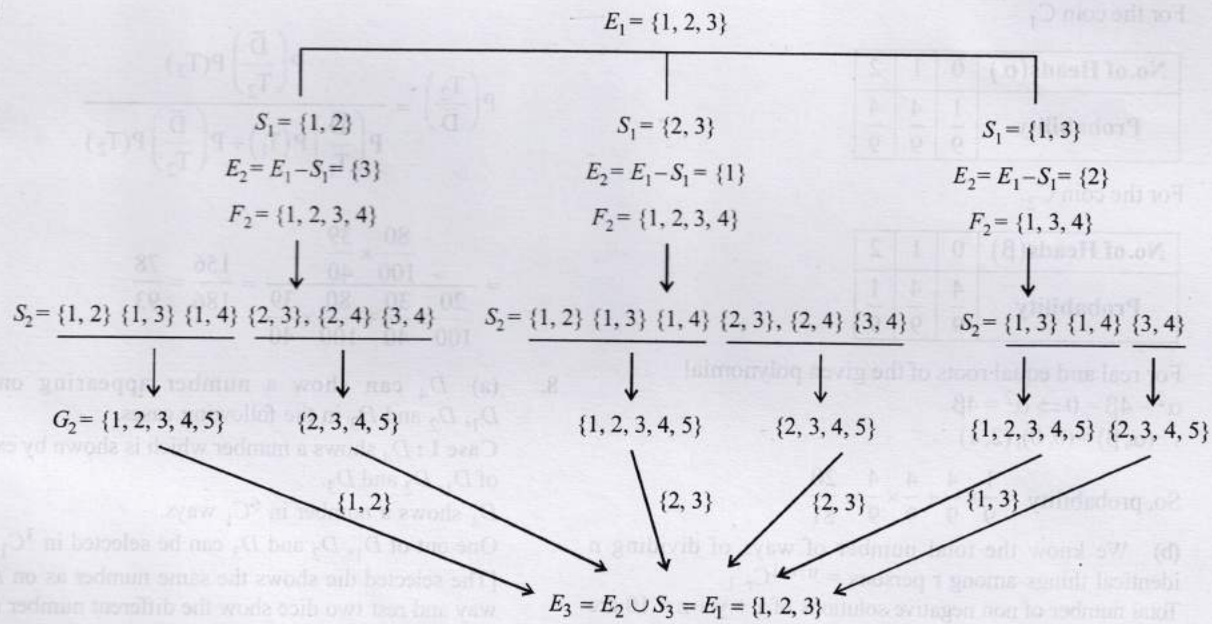
$$P\left(\frac{E_1}{F}\right) = \frac{P(E_1)\times P\left(\frac{F}{E_1}\right)}{P(E_1)\times P\left(\frac{F}{E_1}\right) + P(E_2)\times P\left(\frac{F}{E_2}\right)}$$

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{6}{(n+5)(n+4)}} = \frac{6}{11}$$

$$11(n^2+9n+20) = 6(n^2+9n+80)$$

$$= 5n^2 + 45n - 260 = 0 \Rightarrow n = 4$$

4. (a)



$$P = \frac{P(S_1 \cap (E_1 = E_3))}{P(E_1 = E_3)} = \frac{P(A_{1,2})}{P(A)}$$

$$P(A) = P(A_{1,2}) + P(A_{1,3}) + P(A_{2,3})$$

↑ If 1,2 chosen at start; ↑ If 1,3 chosen at start; ↑ If 2,3 chosen at start

$$P(A_{1,2}) = \frac{1}{3}\times\underbrace{\frac{1\times{}^3C_1}{{}^4C_2}}_{\text{1 is definitely chosen from } F_2}\times\underbrace{\frac{1}{{}^5C_2}}_{\text{1, 2 chosen from } G_2}$$

$$= \frac{1}{3}\times\frac{1}{2}\times\frac{1}{10} = \frac{1}{60}$$

$$P(A_{1,3}) = \frac{1}{3}\times\underbrace{\frac{1\times{}^2C_1}{{}^3C_2}}_{\text{1 is definitely chosen from } F_2}\times\underbrace{\frac{1}{{}^5C_2}}_{\text{1, 2 chosen from } G_2}$$

$$= \frac{1}{3}\times\frac{2}{3}\times\frac{1}{10} = \frac{1}{45}$$

$$P(A_{2,3}) = \frac{1}{3}\times\left[\underbrace{\frac{{}^3C_2\times 1}{{}^4C_2}}_{\text{If 1 is not chosen from } F_2}\times\frac{1}{{}^4C_2} + \underbrace{\frac{1\times{}^3C_1}{{}^4C_2}}_{\text{If 1 is chosen from } F_2}\times\frac{1}{{}^5C_2}\right]$$

$$=\frac{1}{3}\left(\frac{1}{12}+\frac{1}{20}\right)=\frac{2}{45}$$

$$P(A)=\frac{1}{60}+\frac{1}{45}+\frac{2}{45}=\frac{1}{12}$$

$$\frac{P(A_{1,2})}{P(A)}=\frac{1}{5}$$

5. (b) Probability of getting head on coin $C_1 = P(H) = \frac{2}{3}$

Probability of getting head on coin $C_2 = P(H) = \frac{1}{3}$

For the coin C_1

No.of Heads(α)	0	1	2
Probability	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{4}{9}$

For the coin C_2.

No.of Heads(β)	0	1	2
Probability	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

For real and equal roots of the given polynomial
$\alpha^2-4\beta=0 \Rightarrow \alpha^2=4\beta$
$\therefore (\alpha,\beta)=(0,0),(2,1)$

So, probability $=\frac{1}{9}\times\frac{4}{9}+\frac{4}{9}\times\frac{4}{9}=\frac{20}{81}$

6. (b) We know the total number of ways of dividing n identical things among r persons $= {}^{n+r-1}C_{r-1}$
Total number of non negative solutions of $x+y+z=10$ are $={}^{12}C_2=66$
If z is even then there can be following cases arise:
$z=0 \Rightarrow$ No. of ways of solving $x+y=10 \Rightarrow {}^{11}C_1$
$z=2 \Rightarrow$ No. of ways of solving $x+y=8 \Rightarrow {}^{9}C_1$
$z=4 \Rightarrow$ No. of ways of solving $x+y=6 \Rightarrow {}^{7}C_1$
$z=6 \Rightarrow$ No. of ways of solving $x+y=4 \Rightarrow {}^{5}C_1$
$z=8 \Rightarrow$ No. of ways of solving $x+y=2 \Rightarrow {}^{3}C_1$
$z=10 \Rightarrow$ No. of ways of solving $x+y=0 \Rightarrow 1$
$\therefore$ Total ways when z is even $=11+9+7+5+3+1=36$

$\therefore$ Required probability $=\frac{36}{66}=\frac{6}{11}$

7. (c) Given $P(T_1)=\frac{20}{100}, P(T_2)=\frac{80}{100}, P(D)=\frac{7}{100}$

Let $P\left(\frac{D}{T_2}\right)=P$, then $P\left(\frac{D}{T_1}\right)=10P$

By total probability,

$$P(D)=P(T_1)\,P\left(\frac{D}{T_1}\right)+P(T_2)P\left(\frac{D}{T_2}\right)$$

$$\Rightarrow \frac{7}{100}=\frac{20}{100}\times 10P+\frac{80}{100}\times P$$

$$\Rightarrow \frac{7}{280}=P \Rightarrow P=\frac{1}{40}$$

$$\therefore P\left(\frac{D}{T_1}\right)=\frac{10}{40} \text{ and } P\left(\frac{D}{T_2}\right)=\frac{1}{40}$$

$$\Rightarrow P\left(\frac{\bar{D}}{T_1}\right)=1-\frac{10}{40}=\frac{30}{40} \text{ and } P\left(\frac{\bar{D}}{T_2}\right)=1-\frac{1}{40}=\frac{39}{40}$$

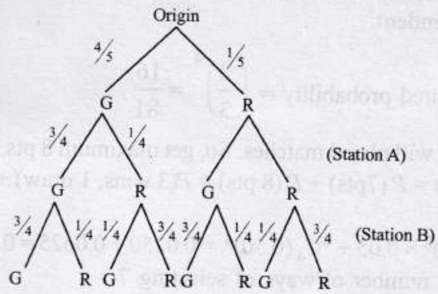
$$P\left(\frac{T_2}{\bar{D}}\right)=\frac{P\left(\frac{\bar{D}}{T_2}\right)P(T_2)}{P\left(\frac{\bar{D}}{T_1}\right)P(T_1)+P\left(\frac{\bar{D}}{T_2}\right)P(T_2)}$$

$$=\frac{\frac{80}{100}\times\frac{39}{40}}{\frac{20}{100}\times\frac{30}{40}+\frac{80}{100}\times\frac{39}{40}}=\frac{156}{186}=\frac{78}{93}$$

8. (a) D_4 can show a number appearing on one of D_1, D_2 and D_3 in the following cases.
Case I : D_4 shows a number which is shown by exactly one of D_1, D_2 and D_3.
D_4 shows a number in ${}^6C_1$ ways.
One out of D_1, D_2 and D_3 can be selected in ${}^3C_1$ ways.
[The selected die shows the same number as on D_4 in one way and rest two dice show the different number in 5 ways each.]
$\therefore$ Number of ways
$={}^6C_1\times{}^3C_1\times 1\times 5\times 5=450$
Case II : D_4 shows a number which is shown by exactly two of D_1, D_2 and D_3.
Number of ways
$={}^6C_1\times{}^3C_2\times 1\times 1\times 5=90$
Case III : D_4 shows a number which is shown by all three dice D_1, D_2 and D_3.
Number of ways
$={}^6C_1\times{}^3C_3\times 1\times 1\times 1=6$
$\therefore$ Total number of favourable ways $=450+90+6=546$
Total ways $=6\times 6\times 6\times 6$

$\therefore$ Required Probability $=\frac{546}{6\times6\times6\times6}=\frac{91}{216}$

9. (c) From the tree diagram.



P (original signal is green / signal received at B is green)

$$=\frac{P(GGG)+P(GRG)}{P(GGG)+P(GRG)+P(RGG)+P(RRG)}$$

$$=\frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}}$$

$$=\frac{\frac{4}{5}\times\frac{10}{16}}{\frac{4}{5}\times\frac{10}{16}+\frac{1}{5}\times\frac{6}{16}}=\frac{40}{40+6}=\frac{40}{46}=\frac{20}{23}$$

10. (c) Given $\omega^{r_1}+\omega^{r_2}+\omega^{r_3}=0$. If ω is a complex cube root of unity then,

Sum of consecutive power ω is zero

$\omega^{3m}+\omega^{3m+1}+\omega^{3m+2}=0$

where m, is integer.

r_1, r_2, r_3 are the numbers obtained on die, these can take any value from 1 to 6.

$\therefore$ m can take values 1 or 2 for r_1, values 0 or 1 for r_2 and values 0 or 1 for r_3

$\therefore$ Number of ways of selecting r_1, r_2, r_3

$= {}^2C_1\times{}^2C_1\times{}^2C_1\times 3!.$

Also the total number of ways of getting r_1, r_2, r_3 on die $= 6\times 6\times 6$

$\therefore$ Required probability $=\frac{{}^2C_1\times{}^2C_1\times{}^2C_1\times 3!}{6\times6\times6}=\frac{2}{9}$

11. (d) Given that A and B to be independent events

$\therefore$ $P(A\cap B)=P(A)\,P(B)\Rightarrow\frac{n(A\cap B)}{n(S)}=\frac{n(A)}{n(S)}\times\frac{n(B)}{n(S)}$

$\Rightarrow\frac{n(A\cap B)}{10}=\frac{4}{10}\times\frac{a}{10}\Rightarrow n(A\cap B)=\frac{5}{2}y$

$\Rightarrow$ $n(A\cap B)$ has to be integer, we have $b = 5$ or 10

$\therefore$ $n(B) = 5$ or 10

12. (c) Given that E, F, G are pairwise independent events.

$$=P(E^c\cap F^c/G)=\frac{P\left(\left(E^c\cap F^c\right)\cap G\right)}{P(G)}$$

$$=\frac{P\left(\left(E\cup F\right)^c\cap G\right)}{P(G)}=\frac{P(G)-P((E\cup F)\cap G)}{P(G)}$$

$$=\frac{P(G)-P((E\cap G)\cup(F\cap G))}{P(G)}$$

$$=\frac{P(G)-P(E\cap G)-P(F\cap G)+P(E\cap F\cap G)}{P(G)}$$

$$=\frac{P(G)-P(E)-P(G)-P(F)\cdot P(G)+0}{P(G)}$$

$$=\frac{P(G)-P(E)-P(G)-P(F)\cdot P(G)+0}{P(G)}$$

$[\because P(E\cap F\cap G)=0]$

$$=\frac{P(G)[1-P(E)-P(F)]}{P(G)}$$

$=P(E^c)-P(F).$

13. (c) Let $E\equiv$ The Indian man is seated adjacent to his wife.

$F\equiv$ Each American man is seated adjacent to his wife.

$\because$ 5 couples can be arranged in a circle in 4! ways. But husband and wife can interchange their places in 2! ways.

$\therefore$ Number of ways when all men are seated adjacent to their wives $n(E\cap F)=4!\times(2!)^5$

All 10 persons can be seated in a circle in 9! ways.

$\therefore$ $n(s)=9!$

4 American couples and q Indian husband and wife can be arranged in a circle in 5! ways. But husband and wife can interchange their places in 2 ! ways.

So the number of ways in which each American man is seated adjacent to his wife $= n(F)$.

$=5!\times(2!)^4$

Then $P(E/F)=\frac{P(E\cap F)}{P(F)}$

So $P(E/F)=\frac{(4!\times(2!)^5)/9!}{(5!\times(2!)^4)/9!}=\frac{2}{5}$

14. (a) Probability of getting 1 in single throw of a dice is $P=\frac{1}{6}$ and prob. of not getting 1 is $q=\frac{5}{6}$.

P (getting 1 in even no. of chances)

$=qp+qqqp+qqqqqp+\ldots\infty$

$$= \frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^5\times\frac{1}{6}+\ldots\infty$$

$$= \frac{5}{36}\left[\frac{1}{1-\frac{25}{36}}\right] = \frac{5}{36}\times\frac{36}{11} = \frac{5}{11}$$

15. **(d)** The minimum of two numbers will be less than 4 will be occurs when at least one of the numbers is less than 4.

$\therefore$ P (at least one no. is < 4),

= 1– P (both the no's are $\geq$ 4)

$$= 1-\frac{3}{6}\times\frac{2}{5} = 1-\frac{6}{30} = 1-\frac{1}{5} = 4/5$$

16. **(a)** $N(s) = 100 \times 100$

$\because$ $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, 7^5 = 16807$

$\therefore$ 7^k (where $k \in N$), has unit's digit is 7 or 9 or 3 or 1.

$\therefore$ For $m, n \in N$, $7^m + 7^n$ is divisible by 5 when the units places are (1, 9) or (3, 7).

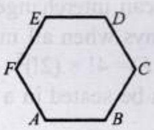
m and n may be selected as follows

	m	n	ways
For (1, 9)	$4r$	$4r+2$	25×25
For (9, 1)	$4r+2$	$4r$	25×25
For (3, 7)	$4r+3$	$4r+1$	25×25
For (7, 3)	$4r+1$	$4r+3$	25×25

$$\therefore \text{Required probability} = \frac{4\times25\times25}{100\times100} = \frac{1}{4}$$

17. **(c)** Total number of triangle = ${}^6C_3$.

Equilateral triangles are ΔACE and ΔBDF.



$$\therefore \text{Required probability} = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

18. **(a)** We have $P[A\cap(B\cup C)] = P[(A\cap B)\cup(A\cap C)]$

$= P(A\cap B) + P(A\cap C) - P(A\cap B\cap C)$

$= P(A)\ P(B) + P(A)\ P(C) - P(A)\ P(B)\ P(C)$

$= P(A)\ [P(B) + P(C) - P(B\cap C)] = P(A)\ P(B\cup C)$

$\therefore$ S_1 is true.

$P(A\cap(B\cap C)) = P(A)\ P(B)\ P(C) = P(A)\ P(B\cap C)$

$\therefore$ S_2 is also true.

19. **(a)** $n(S) = 6$

p (not less than 2 and not greater than 5)

$$= p(2,3,4,5) = \frac{4}{6} = \frac{2}{3}$$

$\because$ The dice is rolled four times and each times results are independent.

$$\therefore \text{Required probability} = \left(\frac{2}{3}\right)^4 = \frac{16}{81}.$$

20. **(b)** India will play 4 matches. So, get maximum 8 pts. P (at least 7 pts) = P (7pts) + P (8 pts) = P(3 wins, 1 draw) + P (4 wins)

$= {}^4C_3\,(0.5)^3\times0.05 + {}^4C_4\,(0.50)^4 = 0.0250 + 0.0625 = 0.0875$

21. **(d)** Total number of ways of selecting 7 coupons out of 15 coupons = 15^7.

Total number of ways of selecting 7 coupons from 1 to 9 numbered coupons = 9^7

Total number of ways of selecting 7 coupons from 1 to 8 numbered coupons = 8^7.

$\therefore$ Number q favourable cases = $9^7 - 8^7$

$$\text{Required probability} = \frac{9^7-8^7}{15^7}.$$

22. **(c)** $P(\bar{A}/\bar{B}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \dfrac{P(\overline{A\cup B})}{P(\bar{B})} = \dfrac{1-P(A\cup B)}{P(\bar{B})}$

23. **(b)** $n = 3, p = 0.4, \Rightarrow q = 0.6$

$\therefore$ P (atleast once) = $P(X\geq1) = 1 - P(X=0)$

$= 1 - {}^3C_0(0.4)^0\,(0.6)^3 = 1 - 0.216 = 0.784$

24. **(a)** We have, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.25 + 0.50 - 0.14 = 0.61$

$\therefore$ $P(A'\cap B') = P((A\cup B)') = 1 - P(A\cup B)$

$= 1 - 0.61 = 0.39$

25. **(d)** The events x and y can happen simultaneously e.g., (4, 5)

$\therefore$ x and y are not mutually exclusive.

Also x and y independent to each other.

26. **(11)** ATQ, N Balls = 3W + 6G + (N – 9) B

$$P(W_1\cap G_2\cap B_3) = \frac{2}{5N}$$

$$\Rightarrow \frac{3}{N}\times\frac{6}{N-1}\times\frac{N-9}{N-2} = \frac{2}{5N}$$

$$\Rightarrow N^2 - 48N + 407 = 0$$

$$\Rightarrow N = 11 \text{ or } 37$$

$$\text{Given, } P(B_3\,|\,W_1\cap G_2) = \frac{2}{9}$$

$$\Rightarrow \frac{P(W_1\cap G_2\cap B_3)}{P(W_1\cap G_2)} = \frac{2}{9}$$

$$\Rightarrow \frac{\frac{2}{5N}}{\frac{3}{N}\times\frac{6}{N-1}} = \frac{29}{9} \Rightarrow \frac{N-1}{45} = \frac{2}{9} \Rightarrow N = 11$$

27. **(31)** Number of five digit numbers divisible by 5

$\underbrace{----}_{1,2,2,2}$ 0 fixed $\rightarrow \frac{\underline{|4}}{\underline{|3}}=4$

$\underbrace{----}_{1,4,2,2}$ 0 fixed $\rightarrow \frac{\underline{|4}}{\underline{|2}}=12$

$\underbrace{----}_{4,2,2,2}$ 0 fixed $\rightarrow \frac{\underline{|4}}{\underline{|3}}=4$ } Total = 38

$\underbrace{----}_{2,2,4,4}$ 0 fixed $\rightarrow \frac{\underline{|4}}{\underline{|2}\,\underline{|2}}=6$

$\underbrace{----}_{1,2,4,4}$ 0 fixed $\rightarrow \frac{\underline{|4}}{\underline{|2}}=12$

Number of five digit numbers divisible by 5 but not by 20

$\underset{2,2,2}{---}$ 1 0 fixed $\rightarrow \frac{\underline{|3}}{\underline{|3}}=1$

$\underset{2,2,4}{---}$ 1 0 fixed $\rightarrow \frac{\underline{|3}}{\underline{|2}}=3$ } Total = 7

$\underset{2,4,4}{---}$ 1 0 fixed $\rightarrow \frac{\underline{|3}}{\underline{|2}}=3$

$\therefore p=\frac{38-7}{38} \therefore 38p=31$

28. **(6)** Let $P(E_1)=x; P(E_2)=y, P(E_3)=z$

$P(\text{only } E_1)=P(E_1\cap \overline{E}_2\cap \overline{E}_3)=x(1-y)(1-z)=\alpha$

$P(\text{only } E_2)=P(\overline{E}_1\cap E_2\cap \overline{E}_3)=(1-x)y(1-z)=\beta$

$P(\text{only } E_3)=P(\overline{E}_1\cap \overline{E}_2\cap E_3)=(1-x)(1-y)z=\gamma$

$P(\text{none})=P(\overline{E}_1\cap \overline{E}_2\cap \overline{E}_3)=(1-x)(1-y)(1-z)=p.$

Now given $(\alpha-2\beta)p=\alpha\beta \Rightarrow x=2y$

and $(\beta-3r)p=2\beta r \Rightarrow y=3z \therefore x=6z$

Hence $\frac{P(E_1)}{P(E_3)}=\frac{x}{z}=6$

29. **(76.25)** Since p_1 = probability that maximum of chosen numbers is at least 81

$p_1=1-$probability that maximum of chosen number is less than or equal to 80

$$p_1=1-\frac{80\times80\times80}{100\times100\times100}=1-\frac{64}{125}$$

$$p_1=\frac{61}{125}$$

$$\therefore \frac{625p_1}{4}=\frac{625}{4}\times\frac{61}{125}=\frac{305}{4}=76.25$$

30. **(24.50)** Since p_2 = probability that minimum of chosen numbers is at most 40

$=1-$probability that minimum of chosen numbers is greater than or equal to 41

$$=1-\frac{60\times60\times60}{100\times100\times100}=1-\frac{27}{125}=\frac{98}{125}$$

$$\therefore \frac{125}{4}p_2=\frac{125}{4}\times\frac{98}{125}=24.50$$

31. **(0.50)**

Total number of 3 × 3 matrices with 0 or $1=2^9=512$

E_2 contains those matrices in which sum of entries is 7.

$\therefore$ It will be contains 7 one's and 2 zeroe's.

$\therefore$ $n(E_2)={}^9C_2=36$

$E_1\cap E_2$ contains those matrices in which 7 ones, 2 zeroes and its det is zero.

Det(A) = 0. This can be occurs when two rows/columns are identical.

e.g.

$$\begin{vmatrix}0&0&1\\1&1&1\\1&1&1\end{vmatrix} \text{ or } \begin{vmatrix}1&0&0\\1&1&1\\1&1&1\end{vmatrix} \text{ or } \begin{vmatrix}0&1&0\\1&1&1\\1&1&1\end{vmatrix}$$

$\therefore$ $n(E_1\cap E_2)={}^3C_1\times{}^3C_1\times2=18$

$$\therefore P(E_1/E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{18/512}{36/512}=\frac{1}{2}=0.50$$

32. **(422)** Let n(A) = a, n(B) = b, $n(A\cap B)=c$

$\therefore$ $1\le b<a$

Also given that A and B are independent events

$\therefore$ $P(A\cap B)=P(A)\,P(B)$

$$\Rightarrow \frac{n(A\cap B)}{n(S)}=\frac{n(A)}{n(S)}\times\frac{n(B)}{n(S)}$$

$$\Rightarrow \frac{c}{6}=\frac{a}{6}\times\frac{b}{6} \Rightarrow ab=6c$$

If a = 6 then b = c = 5, 4, 3, 2, 1 $(\because b<a)$

There is only one way to select all 6 elements of set A.

Number of ways of selecting 5, 4, 3, 2 or 1 elements in B and $A\cap B$ are

${}^6C_5+{}^6C_4+{}^6C_3+{}^6C_2+{}^6C_1=2^6-2=62$

If a = 5 then $b=\frac{6c}{5}$, which is not possible because if c = 5 then b = 6, while b < a.

If a = 4 then $b=\frac{6c}{4}=\frac{3c}{2}$, which is possible because if c = 2 then b = 3

2 elements in $A\cap B$ can be selected in ${}^6C_2$ ways.

2 additional elements in A can be selected in ${}^4C_2$ ways.

1 additional element in B can be selected in ${}^2C_1$ ways.

$\therefore$ No. of ways for a = 4, b = 3, c = 2 are

${}^6C_1\times{}^4C_1\times{}^2C_1=15\times6\times2=180$

If a = 3 then b = 2c $\Rightarrow$ c = 1, b = 2 which can be done in $^6C_1 \times {}^5C_1 + {}^4C_2 = 6 \times 5 \times 6 = 180$ ways.

If a = 2 then b = 3c which is not possible

∴ Total number of required ways
$= 62 + 180 + 180 = 422.$

33. $P(A) = 1 - P(A^c) = 1 - 0.3 = 0.7$

$P(A \cap B^c) = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap B^c) = 0.7 - 0.5 = 0.2$

Now, $P[B \cap (A \cup B^c)] = P[(B \cap A) \cup (B \cap B^c)] = P(A \cap B)$

$$\therefore P[B/(A \cap B^c)] = \frac{P[B \cap (A \cup B^c)]}{P(A \cup B^c)}$$

$$= \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$$

$$= \frac{0.2}{0.7 + 1 - 0.4 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$$

34. Given $P(A \cup B) = 0.8$ and $P(A) = 0.3$

We have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$

[∵ A and B are independent events]

$\Rightarrow 0.8 = 0.3 + P(B) - 0.3\,P(B)$

$\Rightarrow 0.5 = 0.7\,P(B) \Rightarrow P(B) = 5/7$

35. **Case I :** $P[R_A/(R_A, R_B)] = \frac{6}{10} \times \frac{5}{11} \times \frac{6}{10} = \frac{180}{1100} = \frac{18}{110}$

Case II : $P[R_A/(R_A, B_B)] = \frac{6}{10} \times \frac{6}{11} \times \frac{5}{10} = \frac{180}{1100} = \frac{18}{110}$

Case III : $P[R_A/(B_A, R_B)] = \frac{4}{10} \times \frac{4}{11} \times \frac{7}{10} = \frac{56}{550}$

Case IV : $P[R_A/(B_A, B_B)] = \frac{4}{10} \times \frac{7}{11} \times \frac{6}{10} = \frac{168}{1100} = \frac{84}{550}$

∴ The required probability $= \frac{18}{110} + \frac{18}{110} + \frac{56}{550} + \frac{84}{550}$

$$= \frac{90 + 90 + 56 + 84}{550} = \frac{320}{550} = \frac{32}{55}$$

36. Let $P(E_1) = \frac{1+3p}{3}, P(E_2) = \frac{1-p}{4}, P(E_3) = \frac{1-2p}{2}$

Given E_1, E_2 and E_3 are three mutually exclusive events

∴ $P(E_1) + P(E_2) + P(E_3) \le 1$

$\Rightarrow \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1$

$\Rightarrow 4 + 12p + 3 - 3p + 6 - 12p \le 12$

$\Rightarrow 3p \ge 1 \Rightarrow p \ge 1/3$ ... (i)

Now, $0 \le \frac{1+3p}{3} \le 1$

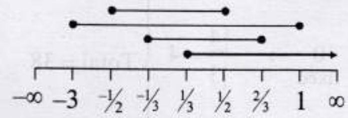
$\Rightarrow 0 \le 1 + 3p \le 3 \Rightarrow -\frac{1}{3} \le p \le \frac{2}{3}$ ... (ii)

And $0 \le \frac{1-p}{4} \le 1$

$\Rightarrow 0 \le 1 - p \le 4 \Rightarrow -3 \le p \le 1$ ... (iii)

And $0 \le \frac{1-2p}{2} \le 1 \Rightarrow -\frac{1}{2} \le p \le \frac{1}{2}$ ... (iv)

From (i), (ii), (iii) and (iv), we get



$\frac{1}{3} \le p \le \frac{1}{2}$

37. Let F ≡ maximum number out of two ≤ 10.

E ≡ minimum number out of two = 5

$n(s) = 100\,C_2, n(F) = 10C_2 = 45$

and $E \cap F = \{(5, 6), (5, 7), (5, 8), (5, 9), (5, 10)\}$

$n(E \cap F) = 5$

$$\therefore P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{5/100C_2}{45/100C_2} = \frac{1}{9}$$

38. **(False)** $P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$= P(A^c) + P(B^c) - P(A^c)\,P(B^c)$

[∵ A and B are independent events]

$= 0.2 + 0.3 - 0.2 \times 0.3 = 0.5 - 0.06 = 0.44 \ne 0.5$

∴ The statement is false.

39. **(False)** Total number of arranging all letters of word

'ASSASSIN' $= \frac{8!}{4!2!} = 840$

A, A, I, N can be arranged in $\frac{4!}{2!} = 12$ ways

$-A-A-I-N-$ Creating 5 places for 4 S.

∴ No two S's occur together in $= {}^5C_4 \times \frac{4!}{2!} = 60$ ways

∴ Req. prob. $= \frac{60}{840} = \frac{1}{14}$ ∴ Statement is False.

40. **(b, d)**

B_1	B_2	B_3
R – 5, G – 5	R – 3, G – 5	R – 5, G – 3

∵ $P(B_1) = \frac{3}{10}, P(B_2) = \frac{3}{10}, P(B_3) = \frac{4}{10}$

$P(G/B_1) = \frac{5}{10}, P(G/B_2) = \frac{5}{8}, P(G/B_3) = \frac{3}{8}$

(a) $P(B_3 \cap G) = P(B_3)\,P(G/B_3) = \frac{4}{10} \times \frac{3}{8} = \frac{3}{20}$

∴ (a) is not true

(b) $P(G/B_3) = \frac{3}{8}$

∴ (b) is true

(c) $\because P(B_3/G)$

$$= \frac{P(G/B_3)P(B_3)}{P(G/B_1)P(B_1)+P(G/B_2)P(B_2)+P(G/B_3)P(B_3)}$$

$$= \frac{\frac{3}{8}\times\frac{4}{10}}{\frac{5}{10}\times\frac{3}{10}+\frac{5}{8}\times\frac{3}{10}+\frac{3}{8}\times\frac{4}{10}} = \frac{\frac{12}{80}}{\frac{15}{100}+\frac{15}{80}+\frac{12}{80}}$$

$$= \frac{12}{80}\times\frac{400}{60+75+60} = \frac{60}{195} = \frac{4}{13}$$

∴ (c) is not true.

(d) $P(G) = P(G/B_1)P(B_1)+P(G/B_2)P(B_2)+P(G/B_3)P(B_3)$

$$= \frac{5}{10}\times\frac{3}{10}+\frac{5}{8}\times\frac{3}{10}+\frac{3}{8}\times\frac{4}{10}$$

$$= \frac{60+75+60}{400} = \frac{195}{400} = \frac{39}{80}$$

∴ (d) is true.

41. **(a, b)** Given that $P(X) = \frac{1}{3}$, $P(X/Y) = \frac{1}{2}$, $P(Y/X) = \frac{2}{5}$

We have $P(X\cap Y) = P(Y/X)\,P(X) = \frac{2}{5}\times\frac{1}{3} = \frac{2}{15}$

∴ (c) is not true.

And $P(Y) = \frac{P(X\cap Y)}{P(X/Y)} = \frac{\frac{2}{15}}{\frac{1}{2}} = \frac{4}{15}$

∴ (a) is true.

$$P(X'/Y) = \frac{P(X'\cap Y)}{P(Y)} = \frac{P(Y)-P(X\cap Y)}{P(Y)}$$

$$= 1 - P(X/Y) = \frac{1}{2}$$

∴ (b) is true.

$$P(X\cup Y) = \frac{1}{3}+\frac{4}{15}-\frac{2}{15} = \frac{7}{15}$$

∴ (d) in not true.

42. **(a, b)**

$$\because P(X/Y) = \frac{P(X\cap Y)}{P(Y)} \Rightarrow \frac{1}{2} = \frac{1/6}{P(Y)} \Rightarrow P(Y) = \frac{1}{3}$$

Similarly, $P(Y/X) = \frac{P(X\cap Y)}{P(X)}$

$$\Rightarrow \frac{1}{3} = \frac{1/6}{P(X)} \Rightarrow P(X) = \frac{1}{2}$$

(a) $P(X\cup Y) = P(X)+P(Y)-P(X\cap Y) = \frac{1}{2}+\frac{1}{3}-\frac{1}{6} = \frac{2}{3}$

∴ (a) is true.

(b) $\because P(X\cap Y) = P(X)P(Y)$

⇒ X and Y are independent events.

∴ (b) is true.

But (c) is not true.

(d) $P(X^C\cap Y) = P(X^C)\times P(Y) = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$

∴ (d) is not true.

43. **(b, d)** Given that $P(X_1) = \frac{1}{2}$, $P(X_2) = \frac{1}{4}$, $P(X_3) = \frac{1}{4}$

$P(X) = P(\text{at least 2 engines are functioning})$

$$= P(X_1\cap X_2\cap X_3^C) + P(X_1\cap X_2^C\cap X_3) + P(X_1^C\cap X_2\cap X_3) + P(X_1\cap X_2\cap X_3)$$

$$= \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{4}$$

(a) $P(X_1^C/X) = \frac{P(X_1^C\cap X)}{P(X)} = \frac{P(X_1^C\cap X_2\cap X_3)}{P(X)}$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

∴ (a) is not true.

(b) P [Exactly two engines are functioning $/X$]

$$= \frac{P[(\text{Exactly two engines are functioning})\cap X]}{P(X)}$$

$$= \frac{P(X_1^C\cap X_2\cap X_3)+P(X_1\cap X_2^C\cap X_3)+P(X_1\cap X_2\cap X_3^C)}{P(X)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$$

∴ (b) is true.

(c) $P(X/X_2) = \frac{P(X\cap X_2)}{P(X_2)}$

$$= \frac{P(X_1\cap X_2\cap X_3)+P(X_1^C\cap X_2\cap X_3)+P(X_1\cap X_2\cap X_3^C)}{P(X_2)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$$

∴ (c) is not true.

(d) $P(X/X_1) = \frac{P(X\cap X_1)}{P(X_1)}$

$$= \frac{P(X_1 \cap X_2 \cap X_3) + P\left(X_1 \cap X_2^C \cap X_3\right) + P\left(X_1 \cap X_2 \cap X_3^C\right)}{P(X_1)}$$

$$= \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$$

∴ (d) is true.

44. **(a, d)** Given that E and F are independent events

∴ $P(E \cap F) = P(E).\ P(F)$ ...(i)

P(exactly one) = $P(E \cap \bar{F}) + P(\bar{E} \cap F) = \frac{11}{25}$

$\Rightarrow P(E)\,P(\bar{F}) + P(\bar{E})\,P(F) = \frac{11}{25}$

$\Rightarrow P(E)(1 - P(F)) + (1 - P(E))\,P(F) = \frac{11}{25}$

$\Rightarrow P(E) - P(E)\,P(F) + P(F) - P(E)\,P(F) = \frac{11}{25}$

$\Rightarrow P(E) + P(F) - 2\,P(E).\,P(F) = \frac{11}{25}$ ...(ii)

P(none of them) = $P(\bar{E} \cap \bar{F}) = \frac{2}{25} \Rightarrow P(\bar{E})\ P(\bar{F}) = \frac{2}{25}$

$\Rightarrow [1 - P(E)]\,[1 - P(F)] = \frac{2}{25}$

$\Rightarrow 1 - P(E) - P(F) + P(E)\,P(F) = \frac{2}{25}$ ...(iii)

Adding equation (ii) and (iii) we get

$1 - P(E)\,P(F) = \frac{13}{25}$ or $P(E)\,P(F) = \frac{12}{25}$ ...(iv)

Using the result in equation (ii) we get

$P(E) + P(F) = \frac{35}{25}$ ...(v)

Solving (iv) and (v) we get

$P(E) = \frac{3}{5}$ and $P(F) = \frac{4}{5}$ or $P(E) = \frac{4}{5}$ and $P(F) = \frac{3}{5}$

∴ (a) and (d) are the correct options.

45. **(b, c)** P (Passing atleast in one subject)

$= P(P \cup C \cup M) = 1 - P\left(\overline{P \cup C \cup M}\right) = 0.75$

$\Rightarrow P.(\bar{P}).P(\bar{C}).P(\bar{M}) = 1 - 0.75 = 0.25 = \frac{1}{4}$

$\Rightarrow (1-m)(1-P)(1-C) = \frac{1}{4}$ ... (i)

P(Passing exactly in two subjects) = 0.4

$\Rightarrow P\left(P \cap C \cap \bar{M}\right) + P\left(P \cap \bar{C} \cap M\right) + P\left(\bar{P} \cap C \cap M\right) = \frac{2}{5}$

$\Rightarrow P.C(1-m) + pm(1-c) + cm(1-p) = \frac{2}{5}$ ...(ii)

P(Passing atleast in two subject) = 0.5

$\Rightarrow Pm(1-c) + Pc(1-m) + cm(1-P) + Pcm = \frac{1}{2}$ ...(iii)

$\Rightarrow pcm = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$ [From (ii)]

∴ (c) is true.

from (i), (ii) and (iii), we get

$p + c + m = \frac{27}{20}$ ∴ (b) is true.

46. **(b)** The no. of ways of placing 3 black balls without any restriction = ${}^{10}C_3$. Now the no. of ways in which no two black balls put together is equal to the no of ways of choosing 3 places marked (–) out of eight places.

– W – W – W – W – W – W – W –

This can be done is ${}^{8}C_3$ ways.

∴ Required probability $= \frac{{}^{8}C_3}{{}^{10}C_3} = \frac{8 \times 7 \times 6}{10 \times 9 \times 8} = \frac{7}{15}$

∴ (b) is the correct option.

47. **(d)** Given : $P(E) \le P(F)$ and $P(E \cap F) > 0$. It conclude that doesn't necessarily mean that E is the subset of F.

∴ The choices (a), (b), (c) do not true in general.

Hence (d) is the right option.

48. **(b)** The probability that only two tests are needed = P (First machine tested is faulty) + P the second machine tested is faulty given the first machine tested is faulty) $= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

49. **(a, d)** We have,

(a) $P(E/F) + P(\bar{E}/F) = \frac{P(E \cap F)}{P(F)} + \frac{P(\bar{E} \cap F)}{P(F)}$

$= \frac{P(E \cap F) + P(\bar{E} \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$

$\left(\because (E \cap F) \cup \left(\bar{E} \cap F\right) = F\right)$

∴ (a) is true.

Also

(b) $P(E/F) + P(E/\bar{F}) = \frac{P(E \cap F)}{P(F)} + \frac{P(E \cap \bar{F})}{P(\bar{F})}$

$= \frac{P(E \cap F)}{P(F)} + \frac{P(E \cap \bar{F})}{1 - P(F)} \neq 1$

∴ (b) does not true. Similarly we can show that (c) does not true.

(d) $P\left(E/\bar{F}\right) + P\left(\bar{E}/\bar{F}\right) = \frac{P\left(E \cap \bar{F}\right)}{P\left(\bar{F}\right)} + \frac{P\left(\bar{E} \cap \bar{F}\right)}{P\left(\bar{F}\right)}$

$$= \frac{P(E \cap \bar{F}) + P(\bar{E} \cap \bar{F})}{P(\bar{F})} = \frac{P(\bar{F})}{P(\bar{F})} = 1.$$

∴ (d) is true.

50. **(a)** P (2 white and 1 black)

$= P(W_1 W_2 B_3 \text{ or } W_1 B_2 W_3 \text{ or } B_1 W_2 W_3)$

$= P(W_1 W_2 B_3) + P(W_1 B_2 W_3) + P(B_1 W_2 W_3)$

$= P(W_1) P(W_2) P(B_3) + P(W_1) P(B_2) P(W_3) + P(B_1) P(W_2) P(W_3)$

[∵ Each are independent event]

$$= \frac{3}{4}.\frac{2}{4}.\frac{3}{4} + \frac{3}{4}.\frac{2}{4}.\frac{1}{4} + \frac{1}{4}.\frac{2}{4}.\frac{1}{4} = \frac{1}{32}(9+3+1) = \frac{13}{32}$$

51. **(c, d)** $P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A) P(B)$

$= P(A') P(B')$ ∴ (c) is true.

Also $P(A \cup B) = P(A) + P(B) - P(A) P(B)$

$\Rightarrow P(A \cap B) = P(A) P(B)$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) P(B)}{P(B)} = P(A)$$

∴ (d) is true.

52. **(a, d)** Let $P(E) = p$ and $P(F) = q$

∵ E and F are independent events

∴ $P(E \cap F) = P(E) P(F)$

$\Rightarrow \frac{1}{12} = pq$ ... (i)

Also $P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$\Rightarrow \frac{1}{2} = 1 - [P(E) + P(F) - P(E) P(F)]$

$\Rightarrow p + q - pq = \frac{1}{2} \Rightarrow p + q = \frac{7}{12}$ ... (ii)

Solving (i) and (ii) we get

either $p = \frac{1}{3}$ and $q = \frac{1}{4}$ or $p = \frac{1}{4}$ and $q = \frac{1}{3}$

∴ (a) and (d) are the correct options.

53. **(a, c)**

(a) We know that $P(A \cup B) \le 1$

$P(A) + P(B) - P(A \cap B) \le 1$

$\Rightarrow P(A \cap B) \ge P(A) + P(B) - 1$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} \ge \frac{P(A) + P(B) - 1}{P(B)} \quad [\because P(B) > 0]$$

$$\Rightarrow P(A/B) \ge \frac{P(A) + P(B) - 1}{P(B)}$$

∴ (a) is correct statement.

(b) We know that

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

∴ (b) is incorrect statement.

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1 - P(\bar{A}) + 1 - P(\bar{B}) - P(A) P(B)$

[∵ A & B are independent events]

$= 2 - P(\bar{A}) - P(\bar{B}) - [1 - P(\bar{A})][1 - P(\bar{B})]$

$= 2 - P(\bar{A}) - P(\bar{B}) - 1 + P(\bar{A}) + P(\bar{B}) - P(\bar{A}) P(\bar{B})]$

$= 1 - P(\bar{A}) P(\bar{B})$ ∴ (c) is the correct statement.

(d) $P(A \cup B) = P(A) + P(B)$ [∵ A and B are disjoint]

∴ (d) is the incorrect statement.

54. **(b, c, d)** Given that E and F are independent event

∴ $P(E \cap F) = P(E) \cdot P(F)$ ...(1)

Now, $P(E \cap F^c) = P(E) - P(E \cap F)$

$= P(E) - P(E) P(F)$ [Using (1)]

$= P(E) [1 - P(F)] = P(E) P(F^c)$

∴ E and F^c are independent. So, (b) in true.

Again $P(E^c \cap F^c) = P(E \cup F)^c = 1 - P(E \cup F)$

$= 1 - P(E) - P(F) + P(E \cap F)$

$= 1 - P(E) - P(F) + P(E) P(F)$

$= ((1 - P(E)(1 - P(F)) = P(E^c) P(F^c)$

∴ E^c and F^c are independent. So, (c) is also true.

Also $P(E/F) + P(E^c/F)$

$$= \frac{P(E \cap F)}{P(F)} + \frac{P(E^c \cap F)}{P(F)} = \frac{P(E)P(F) + P(E^c)P(F)}{P(F)}$$

$$= \frac{P(F)(P(E) + P(E^c))}{P(F)} = 1.$$ So, (d) is also true.

55. **(c)** Given : $P(A \cup B) = 0.6$; $P(A \cap B) = 0.2$

∴ $P(\bar{A}) + P(\bar{B}) = 1 - P(A) + 1 - P(B)$

$= 2 - (P(A) + P(B)) = 2 - [P(A \cup B) + P(A \cap B)]$.

$= 2 - [0.6 + 0.2] = 2 - 0.8 = 1.2$

56. **(c)** Let A, B, C be the events that the student passings test I, II, III respectively.

$P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$

$= P(A) P(B) + P(A) P(C) - P(A) P(B) P(C) = \frac{1}{2}$

$= Pq + P\frac{1}{2} - Pq\frac{1}{2} = \frac{1}{2} \Rightarrow p + pq = 1 \Rightarrow p(1+q) = 1$

which holds for $p = 1$ and $q = 0$.

57. **(a)** Total number of arrangements of seating of 5 students $= 5! = 120$

No. of dearrangements of S_2, S_3, S_4 and S_5 not get peviously seats

$$= 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$$

$= 12 - 4 + 1 = 9$

∴ Required probability $= \frac{9}{120} = \frac{3}{40}$

58. **(c)** Total number of arrangement of seating of 5 students $= 5! = 120$

Favourable cases :

$= \{(S_1 S_3 S_5 S_2 S_4), (S_1 S_4 S_2 S_5 S_3), (S_2 S_4 S_1 S_3 S_5)$

$(S_2 S_5 S_3 S_1 S_4), (S_2 S_4 S_1 S_5 S_3), (S_3 S_1 S_4 S_2 S_5),$

$(S_3\,S_5\,S_1\,S_4\,S_2), (S_3\,S_5\,S_2\,S_4\,S_1), (S_3\,S_1\,S_5\,S_2\,S_4),$
$(S_4\,S_2\,S_5\,S_1\,S_3), (S_4\,S_2\,S_5\,S_3\,S_1), (S_4\,S_1\,S_3\,S_5\,S_2),$
$(S_5\,S_2\,S_4\,S_1\,S_3), (S_5\,S_3\,S_1\,S_4\,S_2)\}$

$\therefore$ Favourable cases = 14

$\therefore$ Required probability $= \frac{14}{120} = \frac{7}{60}$

59. **(a, b)** Let $E_1 \equiv$ box I is selected

$E_2 \equiv$ box II is selected

$F \equiv$ ball drawn is red

$$P(E_2/F) = \frac{P(F/E_2).P(E_2)}{P(F/E_1).P(E_1) + P(F/E_2).P(E_2)}$$

$$= \frac{\frac{n_3}{n_3+n_4}\times\frac{1}{2}}{\frac{n_1}{n_1+n_2}\times\frac{1}{2}+\frac{n_3}{n_3+n_4}\times\frac{1}{2}} = \frac{1}{3}$$

or $\frac{\frac{n_3}{n_3+n_4}}{\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4}} = \frac{1}{3}$

On putting the values of the options we observed that (a) and (b) are the correct options.

60. **(c, d)** $E_1 \equiv$ Red ball is selected from box I

$E_2 \equiv$ Black ball is selected from box I

$F \equiv$ Second red ball is drawn from box I

$\therefore \quad P(F) = P(E_1)\,P(F/E_1) + P(E_2)\,P(F/E_2)$

$$= \frac{n_1}{n_1+n_2}\times\frac{n_1-1}{n_1+n_2-1}+\frac{n_2}{n_1+n_2}\times\frac{n_1}{n_1+n_2-1}$$

On putting the value of the options, we observed that (c) and (d) have the correct values.

61. **(b)** $x_1 + x_2 + x_3$ will be odd. If two of them are even and one is odd or all three are odd.

E_i and O_i denotes the even and odd number resp. from i^{th} box.

$\therefore$ Required probability

$= P(E_1E_2O_3) + P(E_1O_2E_3) + P(O_1E_2E_3) + P(O_1O_2O_3)$

$$= \frac{1}{3}\times\frac{2}{5}\times\frac{4}{7}+\frac{1}{3}\times\frac{3}{5}\times\frac{3}{7}+\frac{2}{3}\times\frac{2}{5}\times\frac{3}{7}+\frac{2}{3}\times\frac{3}{5}\times\frac{4}{7}$$

$$= \frac{8+9+12+24}{105} = \frac{53}{105}$$

62. **(c)** Let x_1, x_2, x_3 are in AP $\Rightarrow 2x_2 = x_1 + x_3$

$\because$ LHS is even, that means x_1 & x_3 can be both even or both odd.

Required probability $= P(E_1\,E_3) + P(O_1, O_3)$

$$= \frac{{}^1C_1\times{}^3C_1+{}^2C_1\times{}^4C_1}{{}^3C_1\times{}^5C_1\times{}^7C_1} = \frac{3+8}{3\times5\times7} = \frac{11}{105}$$

For (Sol. 63–64) : B_1 [1W, 3R, 2B] $\quad B_2$ [2W, 3R, 4B] $\quad B_3$ [3W, 4R, 5B]

63. **(a)** Probability that all three balls are of same colour

= P (RRR) + P (WWW) + P (BBB)

$$= \frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12} = \frac{82}{648}$$

64. **(d)** Let E_1, E_2, E_3 be the events to select bag B_1, B_2 and B_3 respectively.

Let F be the event of getting one white and one red ball.

Then by baye's theorem,

$$P(E_2/F) = \frac{P(F/E_2)P(E_2)}{P(F/E_1)P(E_1)+P(F/E_2)\,P(E_2)+P(F/E_3)P(E_3)}$$

$$= \frac{\frac{1}{3}.\frac{2\times3}{{}^9C_2}}{\frac{1}{3}\left(\frac{1\times3}{{}^6C_2}+\frac{2\times3}{{}^9C_2}+\frac{3\times4}{{}^{12}C_2}\right)} = \frac{55}{181}$$

For (Sol. 65–66) :

$H \rightarrow u_1, (3W, 2W) : u_2\,(1W)$ (1 ball)

$T \rightarrow u_1, (3W, 2W) : u_2\,(1W)$ (2 balls)

65. **(b)** $P(w) = P(H\cap W) + P(T\cap W)$

$= P(H)\,P(W/H) + P(T)\,P(W/T)$

$$= \frac{1}{2}\left\{\frac{3}{5}\times1+\frac{2}{5}\times\frac{1}{2}\right\}+\frac{1}{2}$$

$$\times\left\{\frac{{}^3C_2}{{}^5C_2}\times1+\frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3}+\frac{{}^3C_1.{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right\}$$

$$= \frac{1}{2}\times\frac{8}{10}+\frac{1}{2}\left(\frac{3}{10}+\frac{1}{30}+\frac{12}{30}\right) = \frac{4}{10}+\frac{11}{30} = \frac{23}{30}$$

66. **(d)** $P(H/W) = \frac{P(H\cap W)}{P(W)} = \frac{\frac{1}{2}\left[\frac{3}{5}\times1+\frac{2}{5}\times\frac{1}{2}\right]}{\frac{23}{30}}$

$$= \frac{\frac{4}{10}}{\frac{23}{30}} = \frac{12}{23}$$

67. **(b)** The given system of equastions are

$ax + by = 0, cx + dy = 0$, where $a, b, c, d \in \{0, 1\}$

$\because$ the system of equations have unique solution,

$\therefore \quad \frac{a}{c} \neq \frac{b}{d}$ i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

This condition is satisfy by the following cases -

$\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$\therefore$ Number of favourable cases for the system of equations have unique solution = 6.

Total possible cases of a, b, c, d $\in$ {0, 1} $=2^4 = 16$

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$

$\because$ Statment –1 is true.

$\because$ Homogeneous system of equations always has a solution (Trival solution $x = 0$, $y = 0$ or many solution)

$\therefore$ The probability that the system of equations has a solution is 1.

Hence the statement-2 is true but is not a correct explanation of statement- 1.

68. **(d)** We know that $P(H_i/E) = \frac{P(H_i \cap E)}{P(E)}$

$= \frac{P(E/H_i)P(H_i)}{P(E)} \Rightarrow P(E) = \frac{P(E/H_i)P(H_i)}{P(H_i/E)}$

Now, given that $0 < P(E) < 1$

$\Rightarrow \quad 0 < \frac{P(E/H_i)P(H_i)}{P(H_i/E)} < 1$

$\Rightarrow$ P (E/H$_i$) P (H$_i$) < P (H$_i$/E)

But if $P(H_i \cap E) = 0$ then

P (H$_i$/E) = P (E/H$_i$) = 0

Then P (E/H$_i$) P (H$_i$) < P (H$_i$/E) is not true.

$\therefore$ Statement -1 is not always true.

Also as $H_1, H_2, \ldots H_n$ are mutually exclusive and exhaustive events.

$\therefore P(H_1) + P(H_2) + \ldots + P(H_n) = \sum_{i=1}^{n} P(H_i) = 1$

$\therefore$ Statement -2 is true.

69. Let us consider the events

$E_1 \equiv$ person goes by car,

$E_2 \equiv$ person goes by scooter,

$E_3 \equiv$ person goes by bus,

$E_4 \equiv$ person goes by train,

$F \equiv$ person reaches late

Then according to question

$P(E_1) = \frac{1}{7}; P(E_2) = \frac{3}{7}; P(E_3) = \frac{2}{7}; P(E_4) = \frac{1}{7}$

$P(F/E_1) = \frac{2}{9} \Rightarrow P(\bar{F}/E_1) = 1 - \frac{2}{9} = \frac{7}{9};$

$P(F/E_2) = \frac{1}{9} \Rightarrow P(\bar{F}/E_2) = 1 - \frac{1}{9} = \frac{8}{9}$

$P(F/E_3) = \frac{4}{9} \Rightarrow P(\bar{F}/E_3) = 1 - \frac{4}{9} = \frac{5}{9};$

$P(F/E_4) = \frac{1}{9} \Rightarrow P(\bar{F}/E_4) = 1 - \frac{1}{9} = \frac{8}{9}$

$P\left(E_1/\bar{F}\right)$

$= \frac{P(\bar{F}/E_1).P(E_1)}{P(\bar{F}/E_1).P(E_1) + P(\bar{F}/E_2).P(E_2) + P(\bar{F}/E_3).P(E_3) + P(\bar{F}/E_4).P(E_4)}$

$P(E_1/\bar{F}) = \frac{\frac{7}{9} \times \frac{1}{7}}{\frac{7}{9} \times \frac{1}{7} + \frac{8}{9} \times \frac{3}{7} + \frac{5}{9} \times \frac{2}{7} + \frac{8}{9} \times \frac{1}{7}}$

$= \frac{7}{7 + 24 + 10 + 8} = \frac{7}{49} = \frac{1}{7}$

70. Let us consider the events

$E_1 \equiv$ 4 white balls are drawn in first six draws

$E_2 \equiv$ 5 white balls are drawn in first six draws

$E_3 \equiv$ 6 white balls are drawn in first six draws

$F \equiv$ exactly one white ball is drawn in next two draws (i.e. one white and one red)

Then $P(F) = P(F/E_1)\,P(E_1) + P(F/E_2)\,P(E_2) + P(F/E_3)\,P(E_3)$

$P(F) = P(F/E_1)\,P(E_1) + P(F/E_2)\,P(E_2)$ [$\because P(F/E_3) = 0$]

$= \frac{^{12}C_2 \times {}^6C_4}{^{18}C_6} \cdot \frac{^{10}C_1 \times {}^2C_1}{^{12}C_2} + \frac{^{12}C_1 \times {}^6C_5}{^{18}C_6} \cdot \frac{^{11}C_1 \times {}^1C_1}{^{12}C_2}$

71. $P(A \cup B)\, P(A \cap B)$

$= [P(A) + P(B) - P(A \cap B)]\, P(\bar{A}) \,.\, P(\bar{B})$

$\leq [P(A) + P(B)]\, P(\bar{A}) \,.\, P(\bar{B})$

$= P(A)\, P(\bar{A})\, P(\bar{B}) + P(\bar{A})\, P(B)\, P(\bar{B})$

$= P(A)\,[1 - P(A)]\, P(\bar{B}) + P(\bar{A})\, P(B)\, [1 - P(B)]$

$= P(A)\, P(\bar{B}) - P(A)\, P(A)\, P(\bar{B}) + P(\bar{A})\, P(B)$

$- P(\bar{A})\, P(B)\, P(B)$

$\leq P(A)\, P(\bar{B}) + P(\bar{A})\, P(B) = P(C)$

Thus $P(C) \geq P(A \cup B)\, P(A \cap B)$ is proved.

72. Let us consider the events

$E_1 \equiv A$ hits B, $E_2 \equiv B$ hits A and $E_3 \equiv C$ hits A

Then given $P(E_1) = 2/3$, $P(E_2) = 1/2$ and $P(E_3) = 1/3$

$E \equiv A$ is hit

$P(E) = P(E_2 \cup E_3) = 1 - P(\bar{E}_2 \cap \bar{E}_3)$

$= 1 - P(\bar{E}_2)\,.P(\bar{E}_3) = 1 - \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{3}$

$\therefore P(E_2 \cap \bar{E}_3 / E) = \dfrac{P(E_2 \cap \bar{E}_3)}{P(E)}$

[$\because P(E_2 \cap \bar{E}_3 \cap E) = P(E_2 \cap \bar{E}_3)$ i.e., B hits A and A is hit = B hits A]

$= \dfrac{P(E_2).P(\bar{E}_3)}{P(E)} = \dfrac{1/2 \times 2/3}{2/3} = \dfrac{1}{2}$

73. Let us consider E_1, E_2 and E_3 be events of passing I, II and III exam respectively.

Then a student can qualify the exam in anyone of following ways

Case 1: He passes first and second exam.

Case 2: He passes first, fails in second but passes third exam.

Case 3: He fails in first, passes second and third exam.

$\therefore$ Required probability

$= P(E_1) P(E_2/E_1) + P(E_1) P(\overline{E}_2/E_1) P(E_3/\overline{E}_2) + P(\overline{E}_1) P(E_2/\overline{E}_1) P(E_3/E_2)$

[as an event is dependent on previous one]

$= p.p + p.(1-p).\dfrac{p}{2} + (1-p).\dfrac{p}{2}.p$

$= p^2 + \dfrac{p^2}{2} - \dfrac{p^3}{2} + \dfrac{p^2}{2} - \dfrac{p^3}{2} = 2p^2 - p^3$

74. Let E_1 be the event that the coin drawn is fair and E_2 be the event that the coin drawn is biased.

$\therefore \quad P(E_1) = \dfrac{m}{N}$ and $P(E_2) = \dfrac{N-m}{N}$

F is the event that on tossing the coin the head appears first and then appears tail.

$\therefore P(F/E_1) = \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$ and $P(F/E_2) = \dfrac{2}{3} \times \dfrac{1}{3} = \dfrac{2}{9}$

Now, $P(E_1/F) = \dfrac{P(E_1).P(F/E_1)}{P(E_1).P(F/E_1) + P(E_2).P(F/E_2)}$

$= \dfrac{\frac{m}{N}\left(\frac{1}{4}\right)}{\frac{m}{N}\frac{1}{4} + \frac{N-m}{N}\frac{2}{9}} = \dfrac{m/4}{m/4 + \frac{2(N-m)}{9}} = \dfrac{9m}{m+8N}$

75. The total number of outcomes = 6^n

Number of ways to choose three numbers out of 6 = ${}^6C_3 \times 3^n$. But these include sequences of length n which use exactly two numbers and exactly one number.

$\because$ The number of n-sequences which use exactly two numbers = ${}^3C_2\,[2^n - 1^n - 1^n] = 3\,(2^n - 2)$ and the number of n sequences which are exactly one number = $({}^3C_1)(1^n) = 3$.

Thus, the number of sequences, which use exactly three numbers

$= {}^6C_3\,[3^n - 3\,(2^n - 2) - 3] = {}^6C_3[3^n - 3(2^n) + 3]$

$\therefore$ Required probability, $= \dfrac{{}^6C_3[3^n - 3(2^n) + 3]}{6^n}$

76. Let W_1 and B_1 be the event that a white and a back ball is drawn in the first draw and W be the event that a white ball is drawn in the second draw. Then

$P(W) = P(B_1).P(W/B_1) + P(W_1).P(W/W_1)$

$= \dfrac{n}{m+n}.\dfrac{m}{m+n+k} + \dfrac{m}{m+n}.\dfrac{m+k}{m+n+k}$

$= \dfrac{m(n+m+k)}{(m+n)(m+n+k)} = \dfrac{m}{m+n}$

77. Given that $P(H) = p$

$\therefore \quad P(T) = 1 - p$

Now p_n = prob. that no two or more consecutive heads occur when tossed n times.

$\therefore \quad p_1 = P(H \text{ or } T) = 1$

(Satisfy the condition that no two or more consecutive heads)

Also p_2 = prob. of getting no two or more consecutive heads in 2 times tosses a coin

$= P(HT) + P(TH) + P(TT)$

$= p(1-p) + (1-p)p + (1-p)(1-p) = 1 - p^2$,

p_n = prob. that no two or more consecutive heads occur when tossed n times.

For $n \geq 3$

= P (last out come is T) P (no two or more consecutive heads in $(n-1)$ throw) + P (last out come is H) P ($(n-1)$th throw results in a T) P (no two or more consecutive heads in $(n-2)$ n throws) $= (1-p)\,P_{n-1} + p\,(1-p)\,p_{n-2}$

Hence Proved.

78. The number of ways in which $P_1, P_2, \ldots, P_8$ can be paired in four pairs $= \dfrac{1}{4!} \times {}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 105$

Now, at least players P_1, P_2 and P_3 will certainly reach the second round. And P_4 can reach in final if exactly two players P_1, P_2, P_3 play against each other and remaining player will play against one of the players from P_5, P_6, P_7, P_8 and P_4 plays against one of the remaining three from P_5, P_6, P_7, P_8.

This can be possible in ${}^3C_2 \times {}^4C_1 \times {}^3C_1 = 36$ ways

$\therefore$ Prob. that P_4 and exactly one of $P_5 \ldots P_8$ reach second round $= \dfrac{36}{105} = \dfrac{12}{35}$

If P_1, P_i, P_4 and P_j where $i = 2$ or 3 and $j = 5$ or 6 or 7 reach the second round then they can be paired in 2 pairs in $\dfrac{1}{2!} \times {}^4C_2 \times {}^2C_2 = 3$ ways

But P_4 will reach the final if P_1 plays against P_i and P_4 plays against P_j. Hence the prob. that P_4 reach the final round from the second $= \dfrac{1}{3}$.

$\therefore$ Probability that P_4 reach the final is $\dfrac{12}{35} \times \dfrac{1}{3} = \dfrac{4}{35}$

79. Given that $P(H) = P \Rightarrow P(T) = 1 - P$

$\alpha = P(H) + P(T)P(T)P(T)P(H)$
$+ P(T)\ P(T)P(T)P(T)P(T)P(T)P(H)\ldots\ldots$
$= p + (1-p)^3 p + (1-p)^6 p + \ldots\ldots$

$= p\,[1 + (1-p)^3 (1-p)^6 + \ldots\ldots] = \dfrac{p}{1-(1-p)^3}$ ...(i)

$\beta = P(T)P(H) + P(T)P(T)P(T)P(T)P(H) + \ldots\ldots$

$= (1-p)p + (1-p)^4 p + \ldots\ldots = \dfrac{(1-p)p}{1-(1-p)^3}$ ...(ii)

Eqn (i) and (ii) give expresion for α and β in terms of p.
From (i) and (ii) we get $\beta = (1-p)\alpha$
We have $\alpha + \beta + \gamma = 1$ (exhaustive events and mutually exclusive events)

$\Rightarrow \gamma = 1 - \alpha - \beta = 1 - \alpha - (1-p)\alpha$

$= 1 - (2-p)\ \alpha = 1 - (2-p)\dfrac{p}{1-(1-p)^3}$

$= \dfrac{1-(1-p)^3 - (2p-p^2)}{1-(1-p)^3}$

$= \dfrac{1 - 1 + p^3 + 3p(1-p) - 2p + p^2}{1-(1-p)^3}$

$= \dfrac{p^3 - 2p^2 + p}{1-(1-p)^3} = \dfrac{p(p^2-2p+1)}{1-(1-p)^3} = \dfrac{p(1-p)^2}{1-(1-p)^3}$

80. Given equation is $x^2 + px + q = 0$
∵ Roots of equation is real
∴ $D \geq 0$
$p^2 - 4q \geq 0 \Rightarrow p^2 \geq 4q.$ ...(i)
Following combination of p and q which satisfy eqn. (i).

p	q	No. of digits q can take
2	1	1
3	1, 2	2
4	1, 2, 3, 4	4
5	1, 2, 3, 4, 5, 6	6
6	1, 2, 3, 4, 5, 6,......, 9	9
7	1, 2, 3, 4, 5,......, 10	10
8	1, 2, 3, 4, 5......, 10	10
9	1, 2, 3, 4, 5,......, 10	10
10	1, 2, 3, 4, 5, ..., 10	10
	Total	62

∵ $n(S) = 10 \times 10 = 100$

∴ Required probability $= \dfrac{62}{100} = 0.62.$

81. We have total 14 seats in two vans. And there are 9 boys and 3 girls can be seated in two vans. ∴ The no. of ways of arranging 12 people on 14 seats without restriction is

$^{14}P_{12} = \dfrac{14!}{2!} = 7(13!)$

Now the no. of ways of choosing back seats is 2. And the no. of ways of arranging 3 girls on adjacent seats is 2 (3!).

The number of ways of arranging 9 boys on the remaining 11 seats is $^{11}P_9$
Therefore, the required number of ways

$= 2.(2.3!).\ ^{11}P_9 = \dfrac{4.3!.11!}{2!} = 12!$

Hence, the probability of the required event $= \dfrac{12!}{7.13!} = \dfrac{1}{91}$

82. **Case I :** If the result in coin is head then pair of unbased dice is rolled.

∴ P (getting head) $= P(F_1) = \dfrac{1}{2}$

E = sum is either 7 or 8 = {(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (2, 6), (6, 2), (4, 4), (3, 5), (5, 3)}
∴ P (number is either 7 or 8 when coin shows head)

$= P(E/F_1) = \dfrac{11}{36}$

Case II : If the result in coin is tail then picked one card from eleven cards.

∴ P (getting tail) $= P(F_2) = \dfrac{1}{2}$

∴ P (number either 7 or 8 when coin shows tail)

$= P(E/F_2) = \dfrac{2}{11}$

∴ Required probability
$= P(E) = P(E/F_1)\,P(F_1) + P(E/F_2)\,P(F_2)$

$= \dfrac{1}{2}\left(\dfrac{11}{36} + \dfrac{2}{11}\right) = \dfrac{193}{792}.$

83. Let D = defective and N = non-defective. Then all possible outcomes are {DD, DN, ND, NN}

Also $P(DD) = \dfrac{50}{100} \times \dfrac{50}{100} = \dfrac{1}{4},$

$P(DN) = \dfrac{50}{100} \times \dfrac{50}{100} = \dfrac{1}{4}$ $P(ND) = \dfrac{50}{100} \times \dfrac{50}{100} = \dfrac{1}{4},$

$P(NN) = \dfrac{50}{100} \times \dfrac{50}{100} = \dfrac{1}{4}$

∴ $P(A) = P(DD) + P(DN) = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

∴ $P(B) = P(DN) + P(NN) = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

$P(C) = P(DD) + P(NN) = \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

Now, $P(A \cap B) = P(DN) = \dfrac{1}{4} = P(A).P(B)$

∴ A and B are independent events.

$P(B \cap C) = P(ND) = \dfrac{1}{4} = P(B).P(C)$

∴ B and C are independent events.

$P(C \cap A) = P(DD) = \frac{1}{4} = P(C).\ P(A)$

$\therefore$ C and A are independent events.

$P(A \cap B \cap C) = 0$ (impossible event)

$\neq P(A)\,P(B)\,P(C)$

$\therefore$ A, B, C are dependent events.

Thus we can conclude that A, B, C are pairwise independent but A, B, C are dependent events.

84. Let $E_1 \equiv$ the examinee guesses the answer,

$E_2 \equiv$ the examinee copies the answer

$E_3 \equiv$ the examinee knows the answer,

$F \equiv$ the examinee answers correctly.

Then, $P(E_1) = \frac{1}{3}$; $P(E_2) = \frac{1}{6}$

$\because\ P(E_1) + P(E_2) + P(E_3) = 1$

$\Rightarrow\ P(E_3) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{6-2-1}{6} = \frac{3}{6} = \frac{1}{2}$

$P(F/E_1) = \frac{1}{4}$, $P(F/E_2) = \frac{1}{8}$ and $P(F/E_3) = 1$

$P(E_3/F)$

$$= \frac{P(F/E_3)P(E_3)}{P(F/E_1)P(E_1) + P(F/E_2)P(E_2) + P(F/E_3)P(E_3)}$$

$$= \frac{1.\frac{1}{2}}{\frac{1}{4}.\frac{1}{3} + \frac{1}{8}.\frac{1}{6} + 1.\frac{1}{2}} = \frac{1/2}{\frac{29}{48}} = \frac{1}{2} \times \frac{48}{29} = \frac{24}{29}$$

85. Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$

For each a_i, $1 \le i \le n$, following 4 cases are aries

(i) $a_i \in P$ and $a_i \in Q$ (ii) $a_i \notin P$ and $a_i \in Q$

(iii) $a_i \in P$ and $a_i \notin Q$ (iv) $a_i \notin P$ and $a_i \notin Q$

$\therefore$ Total no. of ways of choosing P and Q is 4^n. Here case (i) is not favourable because $P \cap Q = \phi$

$\therefore$ For each element there are 3 favourable cases and hence total no. of favourable cases $= 3^n$.

Required probability: $(P \cap Q = \phi) = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$.

86. The total number of coins is $N + 7$. $\therefore$ the total number of ways of choosing 5 coins out of $N + 7$ is ${}^{N+7}C_5$. Let E : the sum of the values of the coins is less than one rupee and fifty paise.

$n(\bar{E})$ $\bar{E}$: the total value of the five coins is equal to or more than one rupee and fifty paise.

$= {}^2C_1 \times {}^5C_4 \times {}^NC_0 + {}^2C_2 \times {}^5C_3 \times {}^NC_0 + {}^2C_2 \times {}^5C_2 \times {}^NC_1$

$= 2 \times 5 + 10 + 10N = 10(N+2)$

$\therefore\ P(\bar{E}) = \frac{10(N+2)}{{}^{N+7}C_5} \Rightarrow P(E) = 1 - P(E')$

$= 1 - \frac{10(N+2)}{{}^{N+7}C_5}$

87. $\because$ P(exactly 2 defective) + P(exctly 3 defective)

$= 0.4 + 0.6 = 1$

$\therefore$ These cases are exhaustive and mutually exclusive.

The testing procedure may terminate at the twelth testing in two mutually exclusive ways.

I : When lot contains 2 defective articles.

II : When lot contains 3 defective articles.

Let A_1 be the event that the lot contains 2 defective articles and A_2 the event that the lot contains 3 defective articles. Also let A be the event that the testing procedure ends at the twelth testing.

$\therefore$ Required probability:

$P(A) = P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2)$ ....(i)

Case-I : First 11 draws must contain 10 non-defective and 1 defective article and 12th draw must give a defective article.

$\therefore\ P(A/A_1) = \frac{{}^{18}C_{10} \times {}^2C_1}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{11}{190}$

Case-II : First 11 draws contains 9 non-defective and 2-defective articles and twelth draw contains defective.

$P(A/A_2) = \frac{{}^{17}C_9 \times {}^3C_2}{{}^{20}C_{11}} \times \frac{1}{9} = \frac{11}{228}$

Now substituting the values of $P(A/A_1)$ and $P(A/A_2)$ in eq. (i), we get

$P(A) = 0.4 \times \frac{11}{190} + 0.6 \times \frac{11}{228} = \frac{11}{475} + \frac{11}{380} = \frac{99}{1900}$

88. The total number of ways of ticking the answers

$= {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 - 1 = 15.$

Case I : When A plays 3 games against B.

Required probability = P(correct answer in I chance) + P(correct answer in II chance + P(correct answer in III chance)

$= \frac{1}{15} + \frac{14}{15} \times \frac{1}{14} + \frac{14}{15} \times \frac{13}{14} \times \frac{1}{13} = \frac{1}{5}$

89. Let, $P(A) = \frac{25}{100} = 0.25$, $P(B) = \frac{20}{100} = 0.20$,

$P(A \cap B) = \frac{8}{100} = 0.08$

$\therefore P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.25 - 0.08 = 0.17$

$P(\bar{A} \cap B) = P(B) - P(A \cap B) = 0.20 - 0.08 = 0.12$

Let E be the event that a person reads an advertisement.

According to question $P(E/(A \cap \bar{B})) = \frac{30}{100}$;

$P(E/\bar{A} \cap B) = \frac{40}{100}$

$P(E/AB)=\frac{50}{100}$

$\because$ $(A\cap\overline{B})$, $(\overline{A}\cap B)$ and $(A\cap B)$ are mutually exclusive

$P(E)=P(E/(A\cap\overline{B}))P(A\cap\overline{B})+P(E/\overline{A}\cap\overline{B})$

$P(\overline{A}\cap\overline{B})+P(E/A\cap B).P(A\cap B)$

$=\frac{30}{100}\times0.17+\frac{40}{100}\times0.12+\frac{50}{100}\times0.08$

$=0.051+0.048+0.04=0.139.$

Thus the population that reads an advertisement is 13.9%.

90. Let $P(BC)=x$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)$
$-P(AB)-P(BC)-P(CA)+P(ABC)$

$\Rightarrow P(A\cup B\cup C)=0.3+0.4+0.8$
$-0.08-x-0.28+0.09=1.23-x$

Given $P(A\cup B\cup C)\geq 0.75$ and we have, $P(A\cup B\cup C)\leq 1$

$\therefore\ 0.75\leq P(A\cup B\cup C)\leq 1$

$\Rightarrow\ 0.75\leq 1.23-x\leq 1$

$\Rightarrow\ 0.23\leq x\leq 0.48$

91. Let 2^{nd} ace is obtained in n^{th} drawn and first ace in $(n-1)$ attempts. The probability of drawing one ace in first $(n-1)$ attempts is $\frac{{}^4C_1\times{}^{48}C_{n-2}}{{}^{52}C_{n-1}}$ and other one ace in the n^{th} attempt is, $\frac{{}^3C_1}{[52-(n-1)]}=\frac{3}{53-n}$

Hence the required probability,

$=\frac{4.48!}{(n-2)!(50-n)!}\times\frac{(n-1)!(53-n)}{52!}\times\frac{3}{53-n}$

$=\frac{(n-1)(52-n)(51-n)}{50.49.17.13}$

92. Given that

$P(A)=0.5$ ...(i)

$P(A\cap B)\leq 0.3$ ...(ii)

We have, $P(A)+P(B)-P(A\cap B)=P(A\cup B)\leq 1$

$\Rightarrow\ 0.5+P(B)-P(A\cap B)\leq 1$ [From (i)]

$P(B)\leq 0.5+P(A\cap B)$

$\Rightarrow\ P(B)\leq 0.5+P(A\cap B)\leq 0.5+0.3$ [From (ii)]

$\Rightarrow\ P(B)\leq 0.8\ \therefore P(B)$ can not be 0.9

93. (a) Let us define the events as :

Let E_1, E_2, E_3 and E_4, be the events that the gun hits the target plane, in first, second, third and fourth shot respectively.

Given that, $P(E_1)=0.4$; $P(E_2)=0.3$;

$P(E_3)=0.2$; $P(E_4)=0.1$

$\Rightarrow\ P(\bar{E}_1)=1-0.4=0.6;\ P(\bar{E}_2)=1-0.3=0.7$

$P(\bar{E}_3)=1-0.2=0.8;\ P(\bar{E}_4)=1-0.1=0.9$

$\therefore$ P (at least one shot hits the plane).

$=1-P$ (none of the shots hits the plane)

$=1-P(\bar{E}_1\cap\bar{E}_2\cap\bar{E}_3\cap\bar{E}_4)$

$=1-P(\bar{E}_1).P(\bar{E}_2).P(\bar{E}_3).P(\bar{E}_4)$

[$\because$ All events are independent events]

$=1-0.6\times0.7\times0.8\times0.9=1-0.3024=0.6976$

94. To draw 2 black, 4 white and 3 red balls in order means drawn two black balls at first 2 drawn, 4 white at next 4 drawn, (3rd to 6th drawn) and 3 red at still next 3 drawn (7th to 9th drawn), i.e., $B_1B_2\ W_1W_2W_3W_4R_1\ R_2\ R_3$,

$\therefore$ Required probability

$=\frac{2}{9}\times\frac{1}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3}\times\frac{2}{2}\times\frac{1}{1}=\frac{1}{1260}$

Topic-2: Random Variables, Probability Distribution, Bernoulli Trails, Binomial Distribution, Poisson Distribution

1. **(b)** Given $P(H)=\frac{1}{3}\Rightarrow P(T)=\frac{2}{3}$

Req. prob = P(HH or HTHH or HTHTHH or ....)
+ P(THH or THTHH or THTHTHH or ...)

$=\left(\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}+......\right)$

$+\left(\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}+\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{1}{3}+....\right)$

$=\frac{\frac{1}{3}.\frac{1}{3}}{1-\frac{2}{3}.\frac{1}{3}}+\frac{\frac{2}{3}.\frac{1}{3}.\frac{1}{3}}{1-\frac{2}{3}.\frac{1}{3}}=\frac{5}{21}$

2. **(a)** According to given condition, following cases may arise.

GGBBB, BGGBB,
GBGBB, BGBGB,
GBBGB

Thus favourable cases are = 5 × 2! × 3! = 60

Total ways in which 5 persons can be seated = 5! = 120

$\therefore$ Required probability $=\frac{60}{120}=\frac{1}{2}$

3. **(b)** P (India wins) $=p=1/2$

$\Rightarrow$ P (India loses) $=q=1/2$

Out of 5 matches india's second win occurs at third test

$\Rightarrow$ India wins third test and simultaneously it has won one match from first two and lost the other.

$\therefore$ Required probability $=P(LWW)+P(WLW)$

$=\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^3=\frac{1}{4}$

4. **(d)** Probability of getting head when one coin tossed $=p$

$\Rightarrow$ Probability of tail $=1-p$

We have

$P(X=r)={}^nC_r p^r q^{n-r}$

ATQ, ${}^{100}C_{50}p^{50}(1-p)^{50}={}^{100}C_{51}p^{51}(1-p)^{49}$

$$\Rightarrow \frac{1-p}{p}=\frac{{}^{100}C_{51}}{{}^{100}C_{50}}=\frac{50!50!}{51!49!}=\frac{50}{51} \Rightarrow 51-51p=50p$$

$$\Rightarrow 101p=51 \Rightarrow p=\frac{51}{101}$$

5. **(c)** Probability of a getting a white ball in a single draw

$$=p(w)=\frac{12}{24}=\frac{1}{2}\Rightarrow p(B)=\frac{1}{2}$$

Probability of getting a white ball 4th time in the 7th draw $=P$(getting 3 W in 6 draws) $\times P$(getting W ball at 7th draw)

$$={}^6C_3\left(\frac{1}{2}\right)^3\times\left(\frac{1}{2}\right)^3\times\frac{1}{2}=\frac{5}{32}$$

6. **(42)** Let equation of line is $y=mx+c$

x	0	1	2	3	4	R–{0, 1, 2, 3, 4}
P(x)	c	m + c	2m + c	3m + c	4m + c	0

$$\sum_{x-0}^{4}(mx+c)=1\Rightarrow 10m+5c=1\Rightarrow 2m+c=\frac{1}{5} \quad ...(i)$$

$$\text{Mean}=\Sigma x_iP_i=\sum_{i=0}^{4}(mx_1+c).\,x_1=30m+10c=\frac{5}{2}$$

$$\Rightarrow 3m+c=\frac{1}{4} \quad ...(ii)$$

from (i) and (ii) we get $m=\frac{1}{20}, c=\frac{1}{10}$

$$\Sigma P_ix_i^2=\sum_{i=0}^{4}(mx_1+c)x_1^2$$

$$=\sum_{i=0}^{4}(mx_i^3-cx_i^2)\Rightarrow 100m+30c$$

(Now putting m and c)

$$\Rightarrow \Sigma P_ix_i^2=5+3=8$$

$$\text{Variance}=\Sigma P_ix_i^2-(\Sigma P_ix_i)^2=8-\left(\frac{5}{2}\right)^2=\frac{7}{4}$$

$\therefore 24\alpha=42$

7. **(6)** Given that P = Probability of hit the target $=0.75=\frac{3}{4}$

Z = Probability of miss the target $=1-\frac{3}{4}=\frac{1}{4}$

$\therefore P(X=r)=$ probability of r success $={}^nC_r\left(\frac{3}{4}\right)^r\left(\frac{1}{4}\right)^{n-r}$

$P(X\ge 3)=1-(P(0)+P(1)+P(2))\ge 0.95$

$$\Rightarrow 1-{}^nC_0\left(\frac{1}{4}\right)^n-{}^nC_1\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{n-1}-{}^nC_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^{n-2}\ge 0.95$$

$$\Rightarrow 1-\left(\frac{1+3n+\frac{9n(n-1)}{2}}{4^n}\right)\ge 0.95$$

$$\Rightarrow \frac{2+6n+9n^2-9n}{2.4^n}\ge 1-0.95$$

$$\Rightarrow 9n^2-3n+2\le 0.05\times 4^n\times 2\le\frac{4^n}{10}$$

for $n=5, 212\le 102.4$ (Not true)

for $n=6, 308\le 409.6$ True

Hence least value of $n=6$

8. **(8)** Given that $P(X\ge 2)\ge 0.96$

$\Rightarrow 1-P(X=0)-P(X=1)\ge 0.96$

$\Rightarrow P(X=0)+P(X=1)\le 0.04$

$$\Rightarrow \left(\frac{1}{2}\right)^n+n\left(\frac{1}{2}\right)^n\le 0.04$$

$$\Rightarrow \frac{n+1}{2^n}\le\frac{1}{25}\Rightarrow\frac{2^n}{n+1}\ge 25$$

$\Rightarrow$ Minimum value of n is 8.

9. **(24.00)** Since, P_i = Probability that randomly selected points has i many friends

$P_0=0$ (0 friends); $P_1=0$ (exactly 1 friends)

$P_2=\frac{{}^4C_1}{{}^{49}C_1}=\frac{4}{49}$ (exactly 2 friends)

$P_3=\frac{{}^{20}C_1}{{}^{49}C_1}=\frac{20}{49}$ (exactly 3 friends)

$P_4=\frac{{}^{25}C_1}{{}^{49}C_1}=\frac{25}{49}$ (exactly 4 friends)

x	0	1	2	3	4
P(x)	0	0	$\frac{4}{49}$	$\frac{20}{49}$	$\frac{25}{49}$

$$\text{Mean}=E(x)=\Sigma x_iP_i=0+0+\frac{8}{49}+\frac{60}{49}+\frac{100}{49}=\frac{168}{49}$$

$$7(E(x))=\frac{168}{49}\times 7=24$$

10. **(0.50)** Total number of ways of selecting 2 persons = ${}^{49}C_2$

Number of ways in which 2 friends are selected $= 6 \times 7 \times 2 = 84$

$$\therefore P = \frac{84}{{}^{49}C_2} = \frac{84 \times 2}{49 \times 48} \Rightarrow 7P = \frac{84 \times 2}{49 \times 48} \times 7 = \frac{1}{2}$$

11. **(8.00)** Prime (2, 3, 5, 7, 11) = {(1,1), (1,2), (2,1), (1,4), (2,3), (3,2), (4,1), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), (6,5)}

n(odd prime) = 14

$$\therefore P(\text{odd prime}) = \frac{14}{36}$$

Perfect square = (4,9) = {(1,3), (2,2), (3,1), (3,6), (4, 5), (5, 4), (6, 3)}

n(perfect square) = 7

$$\therefore P(\text{perfect square}) = \frac{7}{36}$$

and P(odd perfect square) $= \frac{4}{36}$

Required probability

$$= \frac{\frac{4}{36} + \frac{14}{36} \times \frac{4}{36} + \left(\frac{14}{36}\right)^2 \frac{4}{36} + \ldots}{\frac{7}{36} + \frac{14}{36} \times \frac{7}{36} + \left(\frac{14}{36}\right)^2 \frac{7}{36} + \ldots} = P = \frac{4}{7}$$

$$\therefore 14P = 14 . \frac{4}{7} = 8$$

12. We have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \; P(A \cap B) = P(A) + P(B) - P(A \cap B)$ $[\because P(A \cup B) = P(A \cap B)]$

$\Rightarrow \; [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)] = 0$

But $P(A), P(B) \geq P(A \cap B)$

$\Rightarrow \; [P(A) - P(A \cap B)] = [P(B) - P(A \cap B)] = 0$

$\Rightarrow \; [P(A) = P(B)] = [P(A \cap B)]$.

13. **(a)** P (atleast one of them solves the problem) $= 1 - P$ (none of them solves it)

$$= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$$

$$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{21}{256} = \frac{235}{256}$$

14. **(a)** The event that the fifth toss results in a head is independent of the event that the first four tosses result in tails.

$\therefore$ Probability of getting head in 5th trial = 1/2.

15. **(a)** Let E: getting 6

$P(X=3)$

$$= P\left(\bar{E} \cap \bar{E} \cap E\right) = P\left(\bar{E}\right).P\left(\bar{E}\right).P(E)$$

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

16. **(b)** $P(X \geq 3) = 1 - [P(X=1) + P(X=2)]$

$$= 1 - \left[\frac{1}{6} + \frac{5}{6} \times \frac{1}{6}\right] = 1 - \frac{11}{36} = \frac{25}{36}$$

17. **(d)** $P(E_1) = P(X \geq 6) = \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6} + \ldots\infty$

$$= \left(\frac{5}{6}\right)^5 \times \frac{1}{6}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots\infty\right] = \left(\frac{5}{6}\right)^5 \times \frac{1}{6} \times \frac{1}{1 - \frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

and $P(E_2) = P\left(X > 3\right) = \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \ldots\infty = \left(\frac{5}{6}\right)^3$

$\because \; E_1 \cap E_2 = X \geq 6 = E_1$

$$\therefore \; P(E_1 / E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{P(E_1)}{P(E_2)} = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

18. **(b)** Given: $P(u_i) \propto i$. Let $P(u_i) = ki$, we have

$\sum P(u_i) = 1$

$$\Rightarrow \sum ki = 1 \Rightarrow k \sum i = 1 \Rightarrow k = \frac{2}{n(n+1)} \Rightarrow P(u_i) = \frac{2i}{n(n+1)}$$

By total probability theorem

$$P(w) = \sum_{i=1}^{n} P(u_i) P(w / u_i) = \sum_{i=1}^{n} \frac{2i}{n(n+1)} \times \frac{i}{n+1}$$

$$= \frac{2}{n(n+1)^2} . \frac{n(n+1)(2n+1)}{6} = \frac{2n+1}{3n+3}$$

$$\left[\because \Sigma n^2 \frac{n(n+1)(2n+1)}{6}\right]$$

$$\therefore \; \lim_{n \to \infty} P(w) = \lim_{n \to \infty} \frac{2n+1}{3n+3} = \lim_{n \to \infty} \frac{2 + 1/n}{3 + 3/n} = \frac{2}{3}$$

19. **(a)** Given that $P(u_i) = c$

By Baye's theorem, $P(u_n/w) = \dfrac{P(w / u_n) P(u_n)}{\sum_{i=1}^{n} P(w / u_i) P(u_i)}$

$$= \frac{c \times \frac{n}{n+1}}{\sum_{i=1}^{n} \frac{i}{n+1}} = \frac{n}{n+1} \times \frac{n+1}{\frac{n(n+1)}{2}} = \frac{2}{n+1}$$

20. (b) $P(w/E)=\dfrac{P(w\cap E)}{P(E)}$

$$=\frac{P(w\cap u_2)+P(w\cap u_4)+...+(w\cap u_n)}{P(u_2)+P(u_4)+...P(u_n)}$$

$$=\frac{\frac{1}{n}\times\frac{2}{n+1}+\frac{1}{n}\times\frac{4}{n+1}+\frac{1}{n}\times\frac{6}{n+1}+...+\frac{1}{n}\times\frac{n}{n+1}}{\frac{1}{n}+\frac{1}{n}+...+\left(\frac{n}{2}\text{ times}\right)}$$

$$=\frac{\frac{2}{n(n+1)}\left[1+2+3...+\frac{n}{2}\right]}{\frac{1}{n}\times\frac{n}{2}}\quad (\because \text{ even})$$

$$=\frac{4}{n(n+1)}\left[\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)}{2}\right]=\frac{n+2}{2(n+1)}$$

21. The total numbers = 100. The numbers, the product of whose digits is 18, are 29, 36, 63 and 92.

$\therefore\quad p=P(E)=\dfrac{4}{100}=\dfrac{1}{25}\Rightarrow q=1-p=\dfrac{24}{25}$ and $n=4$

P (E occuring at least 3 times) = $P(X=3)+P(X=4)$

$$={}^4C_3p^3q+{}^4C_4p^4=4\times\left(\frac{1}{25}\right)^3\left(\frac{24}{25}\right)+\left(\frac{1}{25}\right)^4=\frac{97}{(25)^4}$$

22. **Case I :** When A plays 3 games against B. $P_1=P$(winning two games) + P (winning three games).

$={}^3C_2(0.6)(0.4)^2+{}^3C_3(0.4)^3$

$=0.288+0.064=0.352$

Case II : When A plays 5 games against B.

$P_2=P$ (winning three games) + P (winning four games) + the prob. of winning 5 games

$={}^5C_3(0.6)^2(0.4)^3+{}^5C_4(0.6)(0.4)^4+{}^5C_5(0.4)^5$

$=0.2304+0.0768+0.01024=0.31744$

As $P_1>P_2$

$\therefore$ A must choose the first offer i.e. best of three games.

23. Since given that the man is one step away from starting point means that either (i) man has taken 6 steps forward and 5 steps backward.

or (ii) man has taken 5 steps forward and 6 steps backward.

Consider movement 1 step forward as success and 1 step backward as failure.

$\therefore\quad p$ = Probability of success = 0.4

and q = Probability of failure = 0.6

$\therefore$ Required probability = $P(X=6)+P(X=5)$

$={}^{11}C_6p^6q^5+{}^{11}C_5p^5q^6$

$={}^{11}C_6(0.4)^6(0.6)^5+{}^{11}C_5(0.4)^5(0.6)^6$

$={}^{11}C_6(0.4)^5(0.6)^5(0.4+0.6)$

$=462\times1\times(0.24)^5=0.37$

Hence the required prob. = 0.37